

Computational Assemblies: Analysis, Design, and Fabrication

Peng Song



Ziqi Wang



Marco Livesu

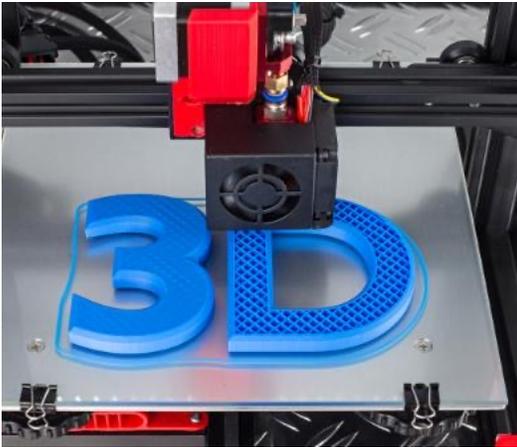


Timetable

		Peng	Ziqi	Marco
	Introduction	~20 mins	X	
	Computational analysis of assemblies	~50 mins	X	
	Computational design of assemblies	~50 mins	X	
	Computational fabrication of assemblies	~50 mins		X
	Q & A	~10 mins	X	X

What Is This Part About?

- Shape decomposition as a mean to **overcome the limitations** of digital fabrication hardware



3D PRINTING



CNC MILLING



LASER CUTTING

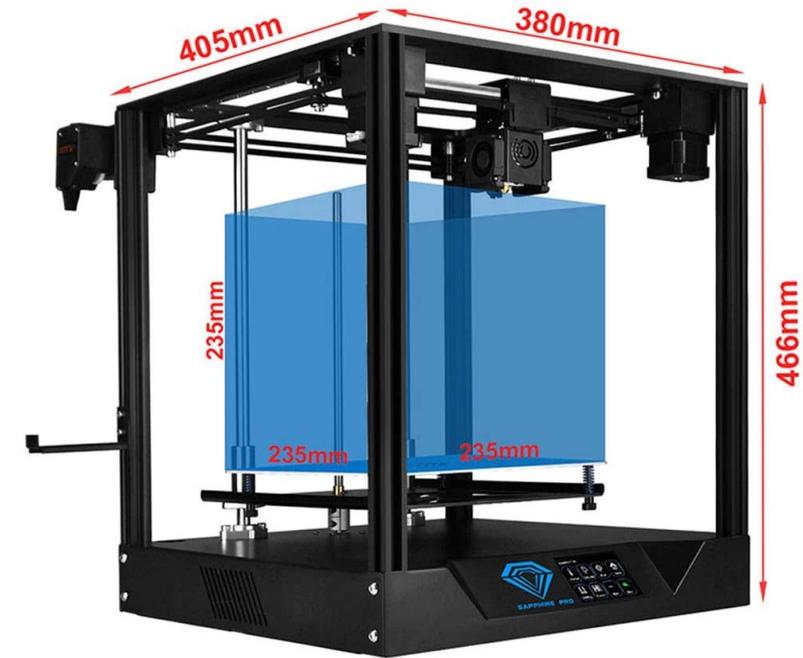
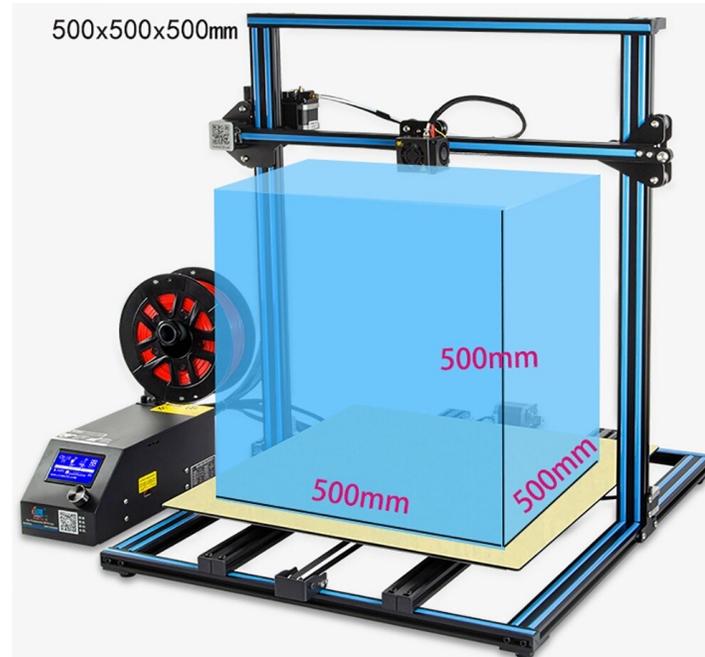


CASTING/MOLDING

Remark #1

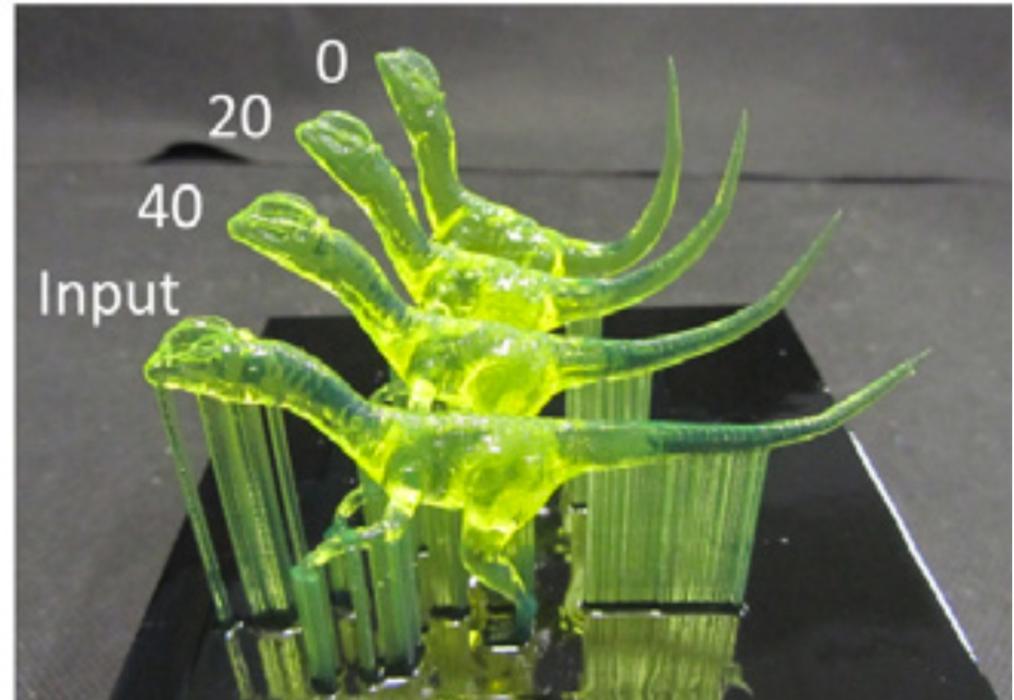
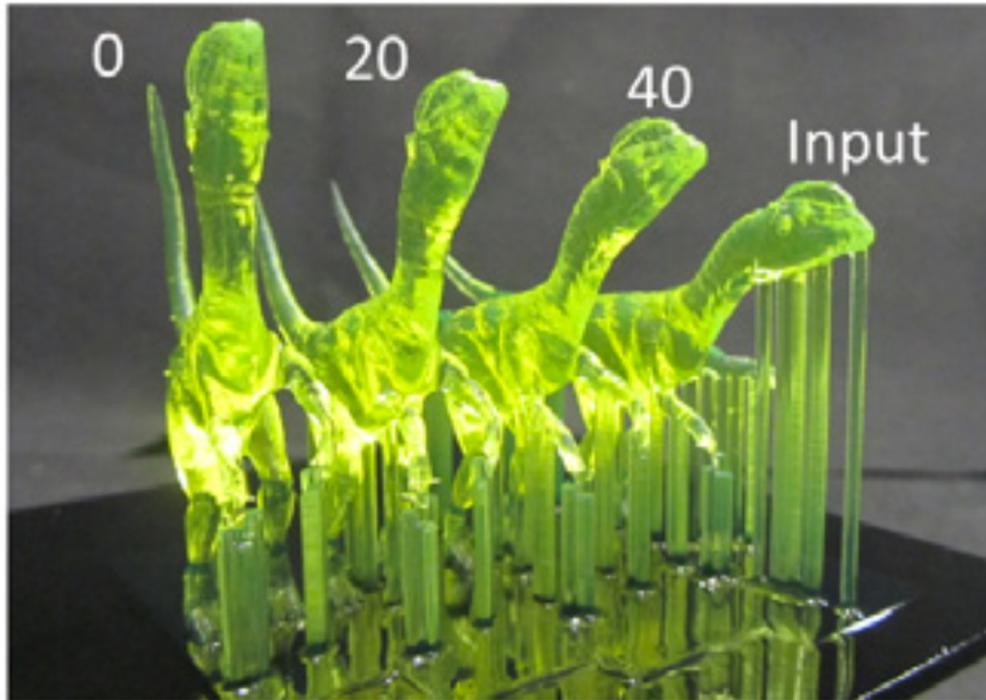
- Unlike previous parts, here assemblies ***are not*** an artistic/functional choice. We split shapes **because we are obliged to!**

- Algorithms must be designed to address **hardware dependent** constraints (e.g. size)
- Constraints can be either **hard** or **soft**



Remark #2

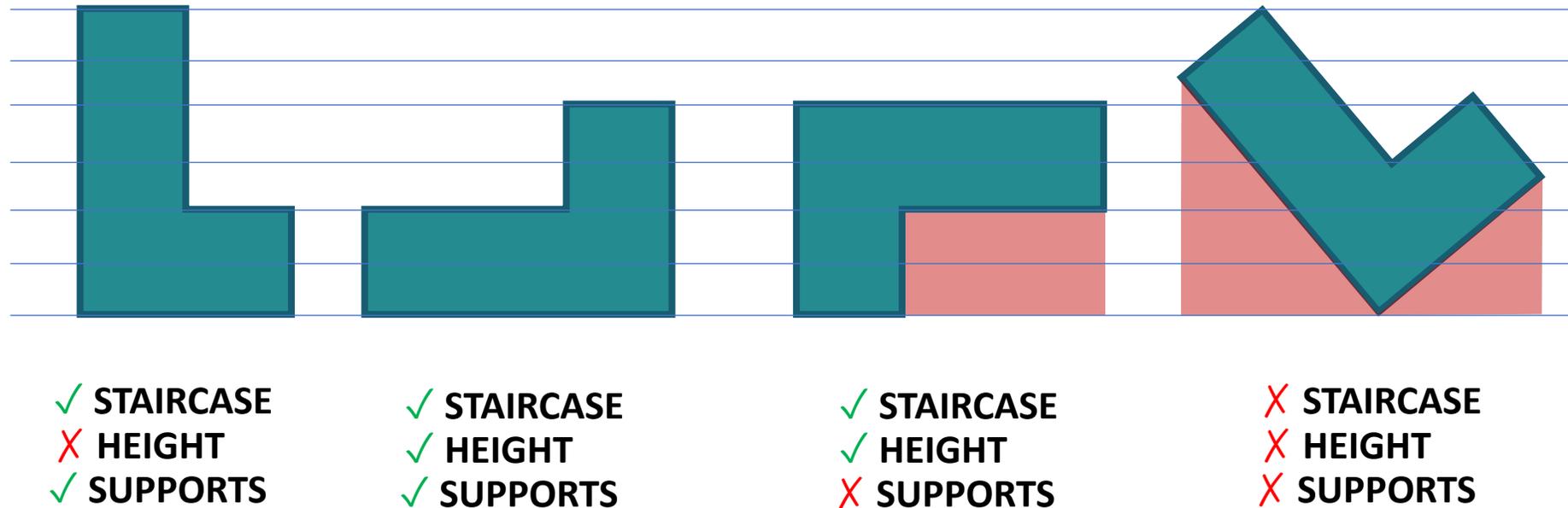
- Splitting is just one possible way to address manufacturing constraints
- Mesh **deformation** is a valid alternative



[Hu et al., CAD 2015]

Remarks

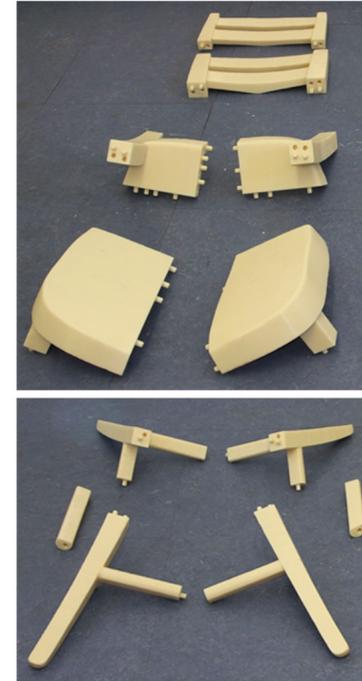
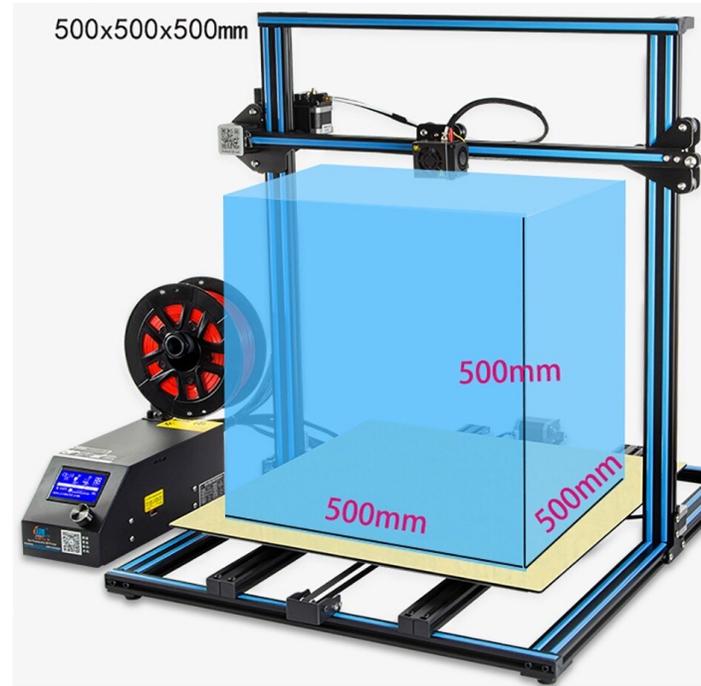
- Splitting is just one possible way to address manufacturing constraints
- Mesh **deformation** is a valid alternative
- Also **re-orientation** may be a valid alternative



[Livesu et al., EG STAR 2017]

Shape Decomposition for Manufacturing

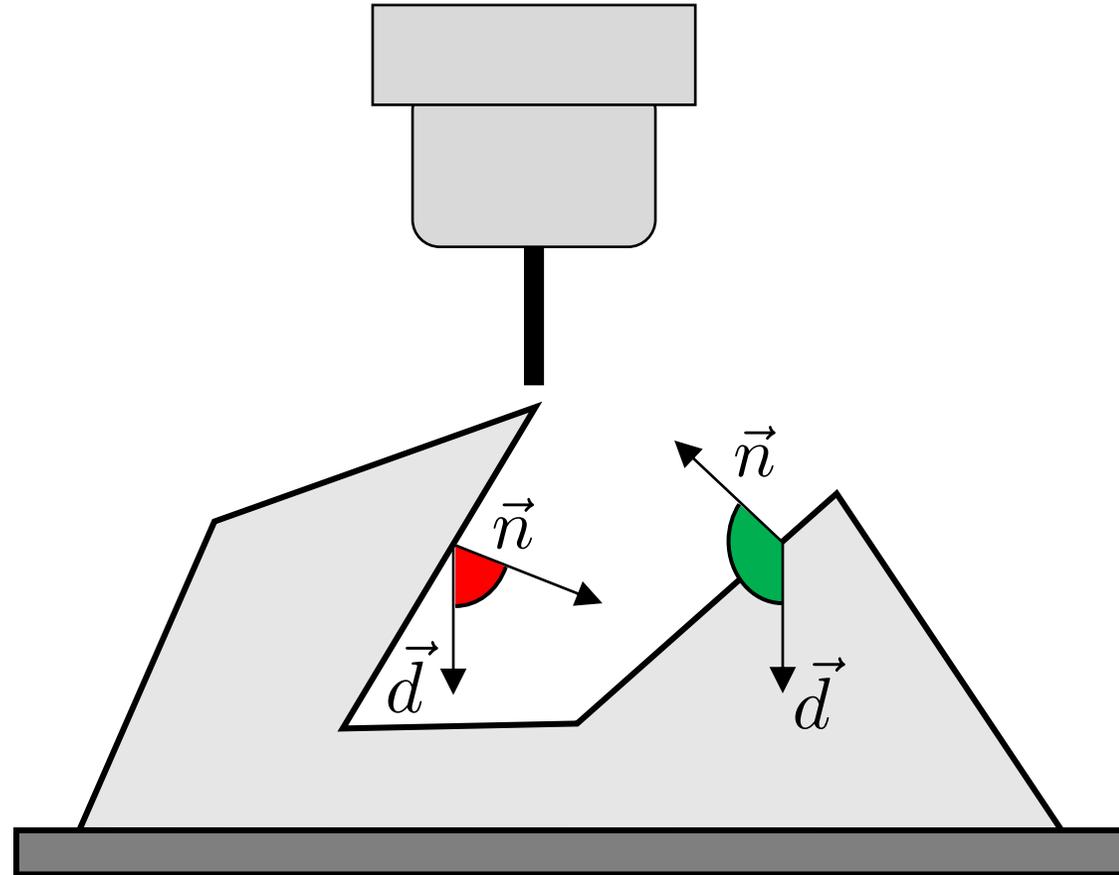
– size



[Luo et al., SIGGRAPH 2012]

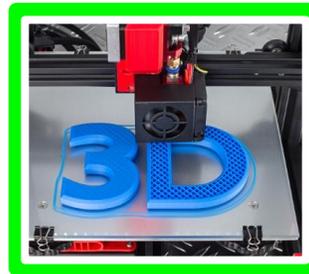
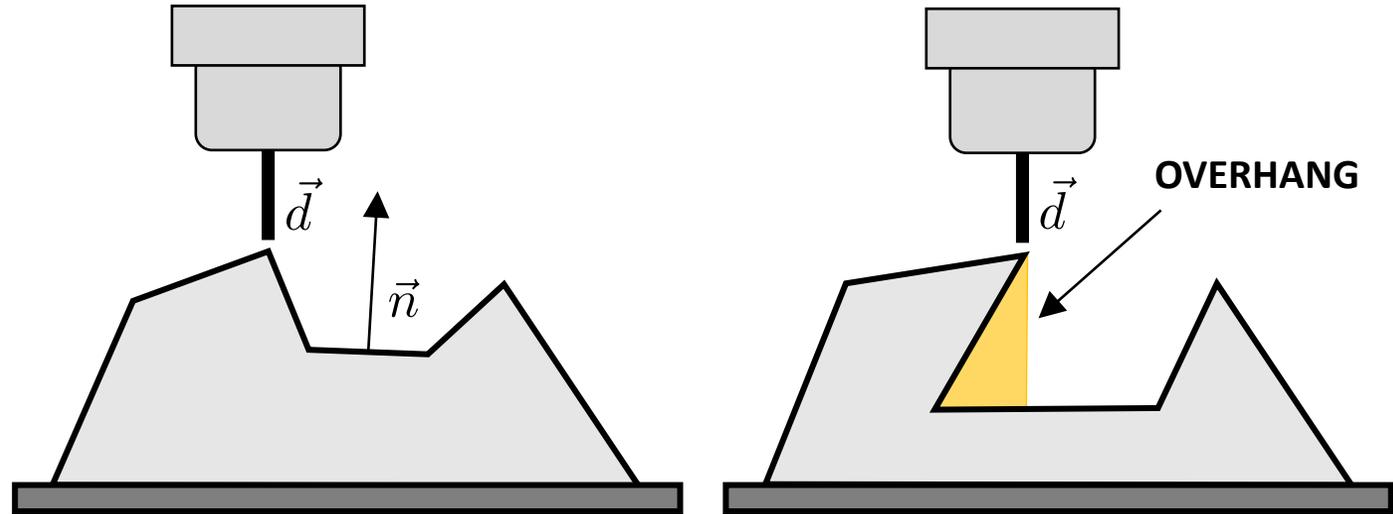
Shape Decomposition for Manufacturing

- size
- **geometry**

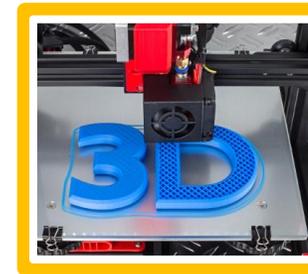


Shape Decomposition for Manufacturing

- size
- **geometry**
 - 3d printing
 - 3 axis milling



no
supports



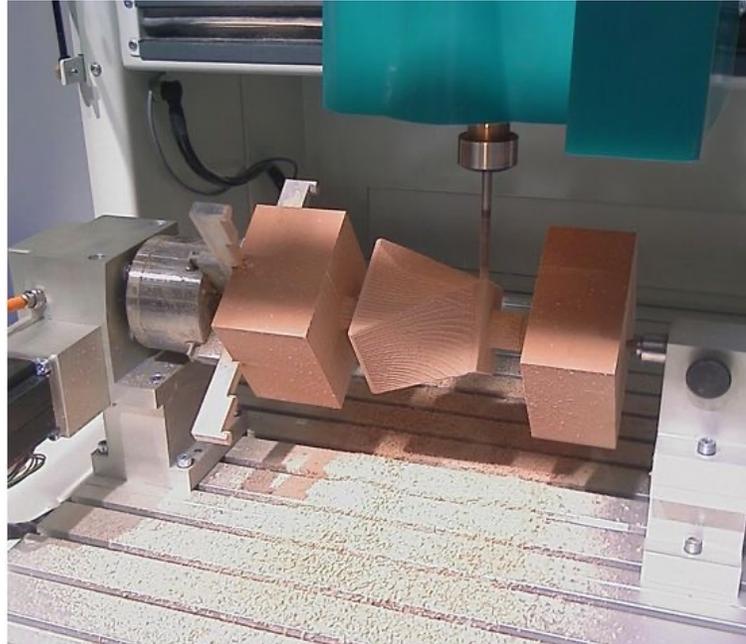
supports
needed



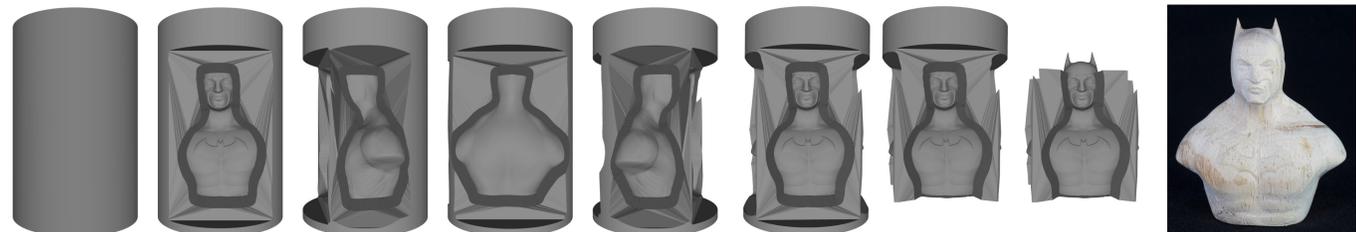
not
fabricable

Shape Decomposition for Manufacturing

- size
- **geometry**
 - 3d printing
 - 3 axis milling
 - **4 axis milling**



[Nuvoli et al., EG 2021]

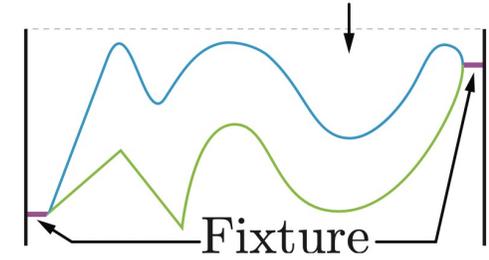
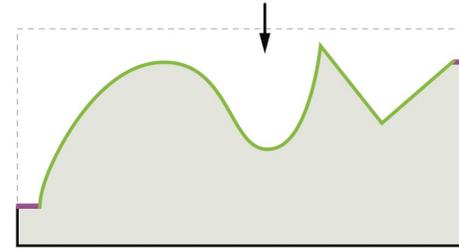
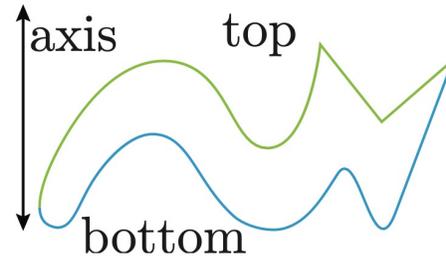


Shape Decomposition for Manufacturing

– size

– **geometry**

- 3d printing
- 3 axis milling
- 4 axis milling
- **DHF**



[Yang et al., SIGGRAPH Asia 2020]

Shape Decomposition for Manufacturing

– size

– **geometry**

- 3d printing
- 3 axis milling
- 4 axis milling
- DHF
- **laser cutting**



[Schüller et al., SIGGRAPH 2018]

Shape Decomposition for Manufacturing

– size

– **geometry**

- 3d printing
- 3 axis milling
- 4 axis milling
- DHF
- laser cutting
- **rigid molding**



[Alderighi et al., SIGGRAPH 2021]

Shape Decomposition for Manufacturing

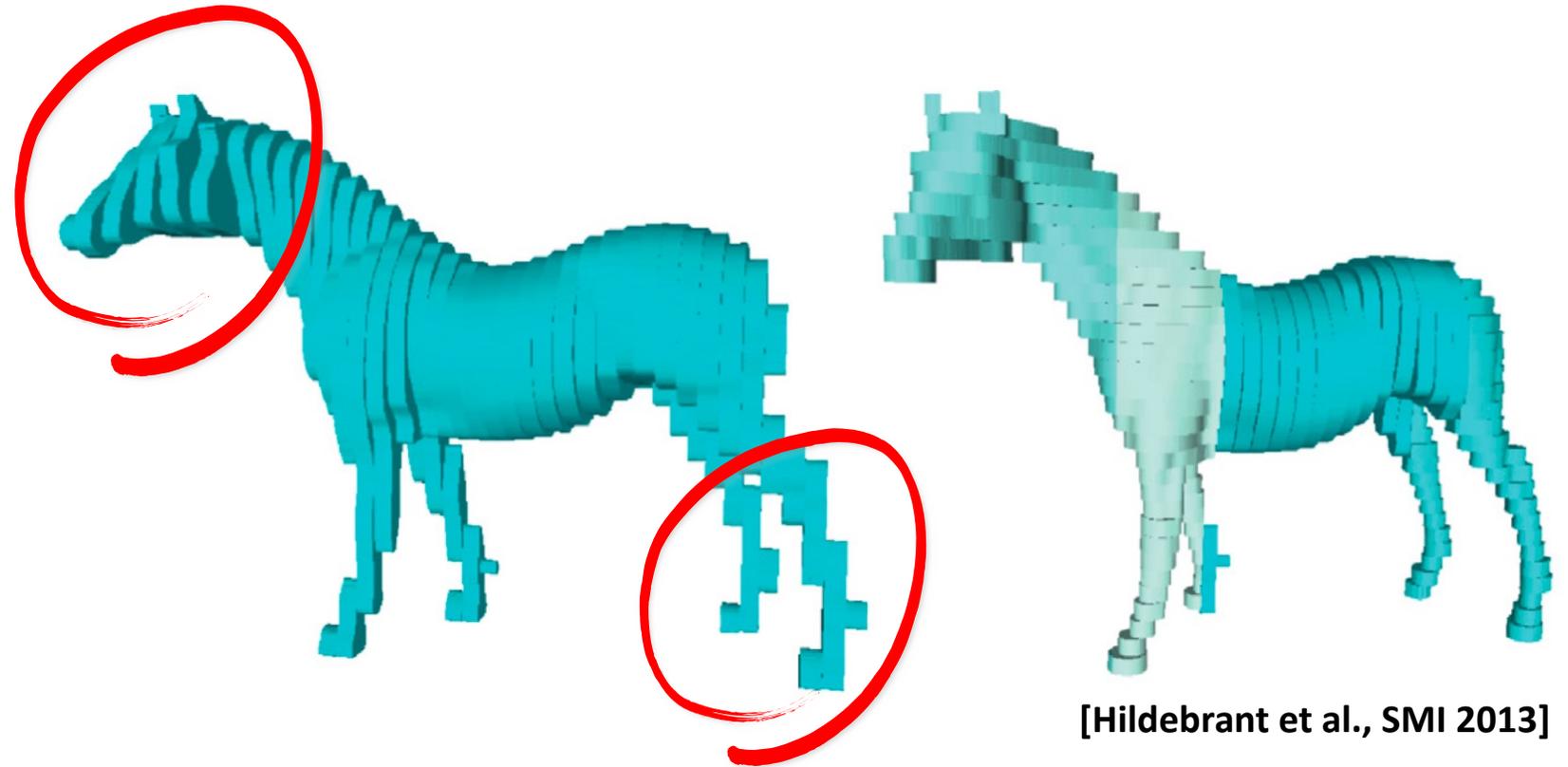
- size
- **geometry**
 - 3d printing
 - 3 axis milling
 - 4 axis milling
 - DHF
 - laser cutting
 - rigid molding
 - **flexible molding**



[Alderighi et al., SIGGRAPH 2019]

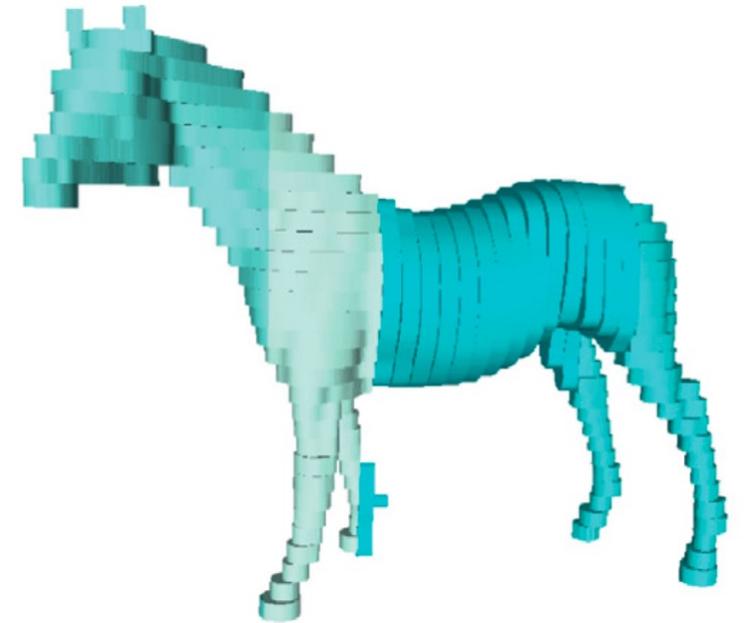
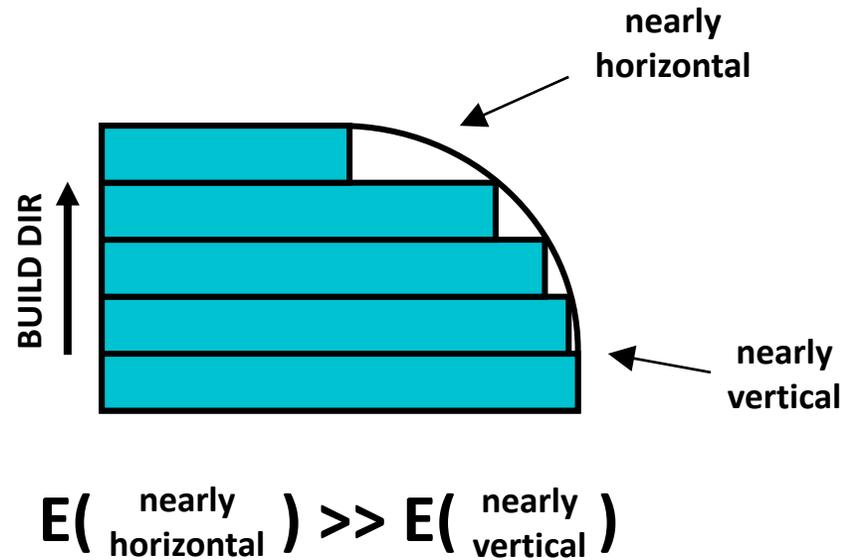
Shape Decomposition for Manufacturing

- size
- geometry
- **staircase effect**



Shape Decomposition for Manufacturing

- size
- geometry
- **staircase effect**



[Hildebrant et al., SMI 2013]

Shape Decomposition for Manufacturing

- size
- geometry
- staircase effect
- **material consumption**

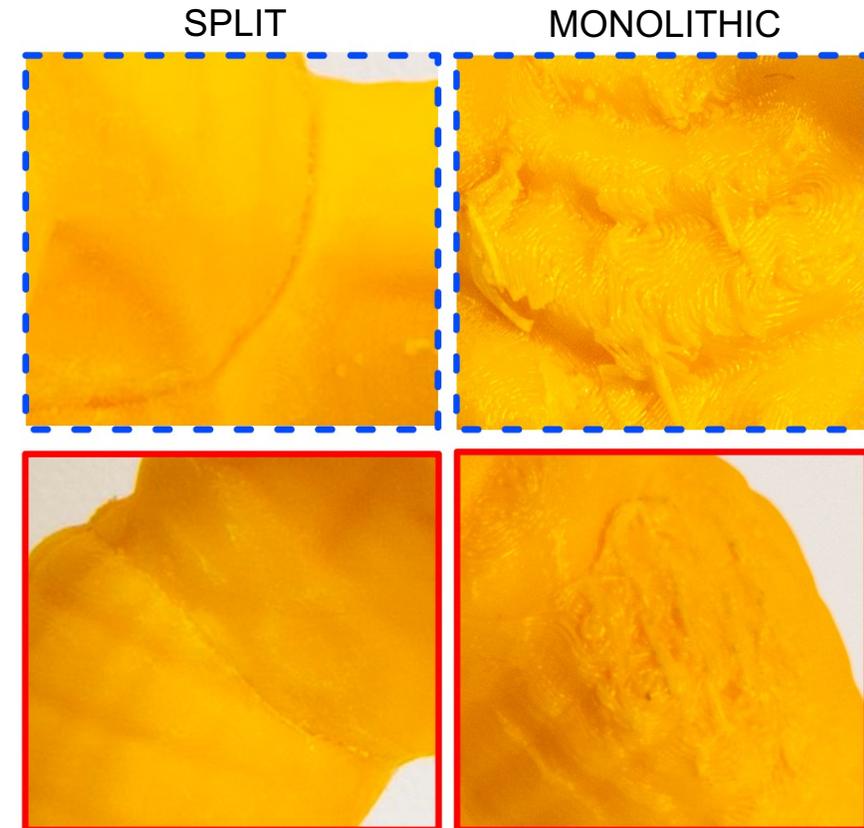


non
pyramidal

pyramidal
decomposition

Shape Decomposition for Manufacturing

- size
- geometry
- staircase effect
- material consumption
- **support artifacts**



Shape Decomposition for Manufacturing

- size
- geometry
- staircase effect
- material consumption
- support artifacts
- **material/color**



single filament

+



composite object

=



homogeneous
assemblable

[Araújo et al., SIGGRAPH 2019]

Shape Decomposition for Manufacturing

- size
- geometry
- staircase effect
- material consumption
- support artifacts
- **material/color**



single filament

+



composite object

=

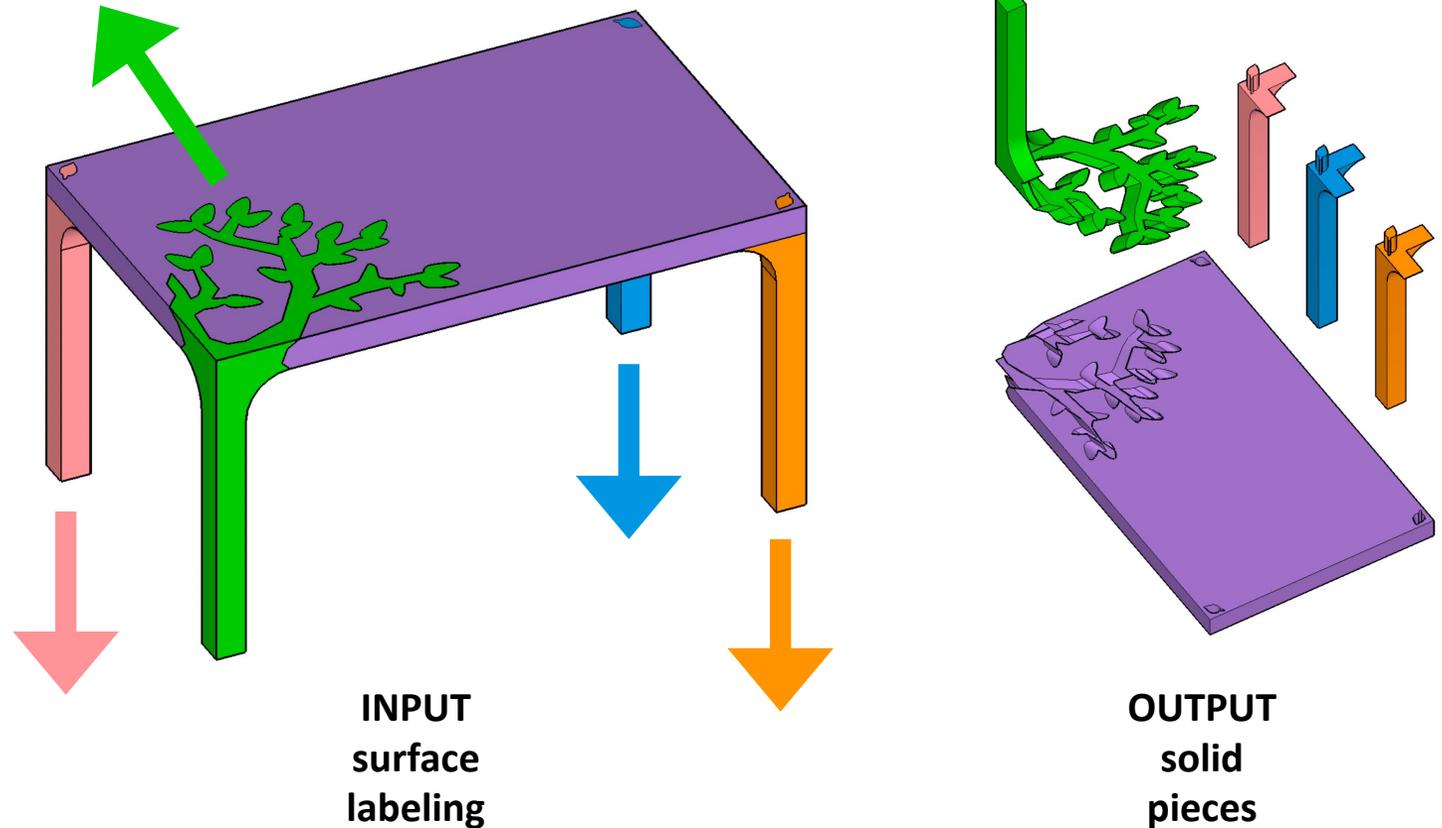


homogeneous
assemblable

[Araújo et al., SIGGRAPH 2019]

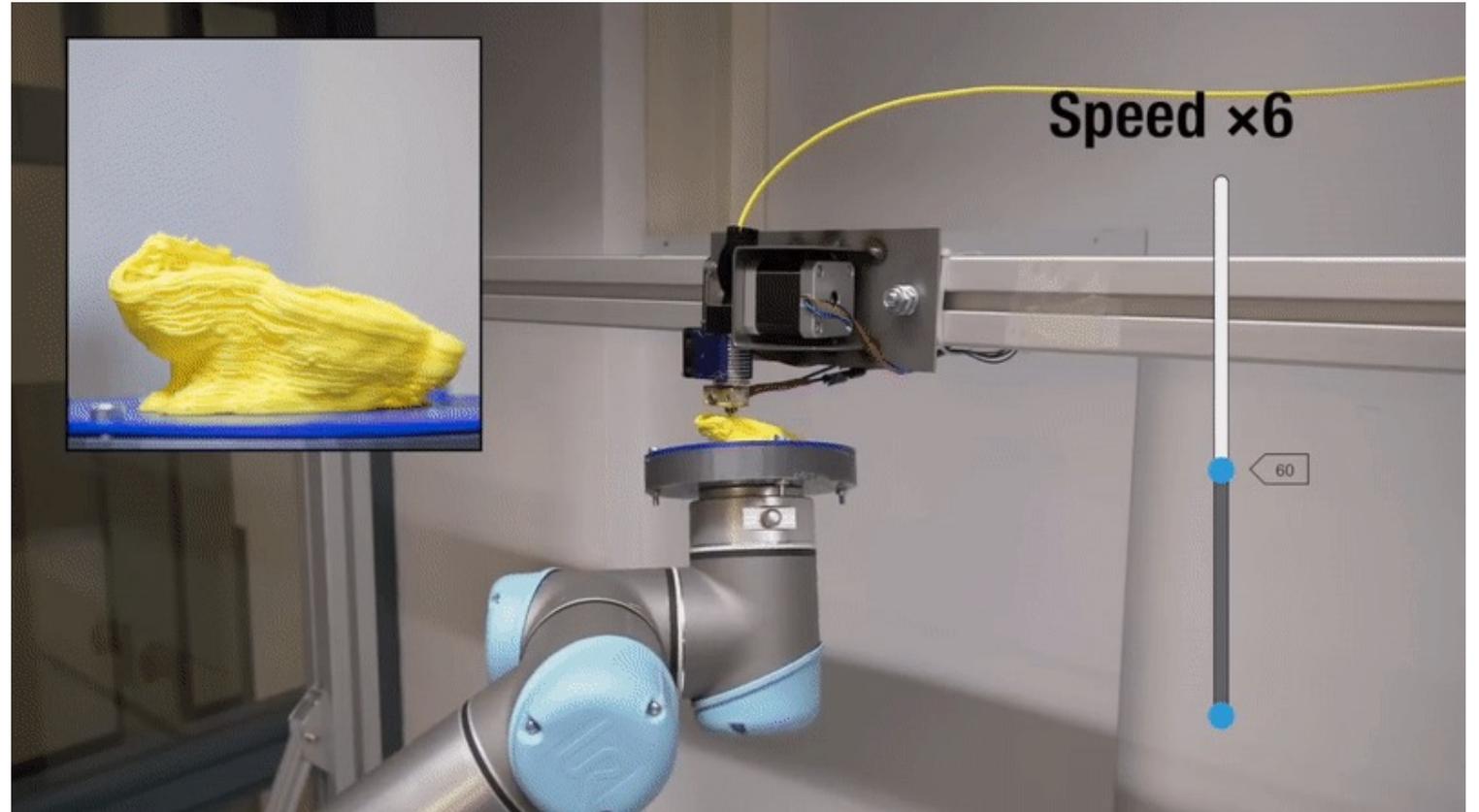
Shape Decomposition for Manufacturing

- size
- geometry
- staircase effect
- material consumption
- support artifacts
- material/color
- **assemblability**



Shape Decomposition for Manufacturing

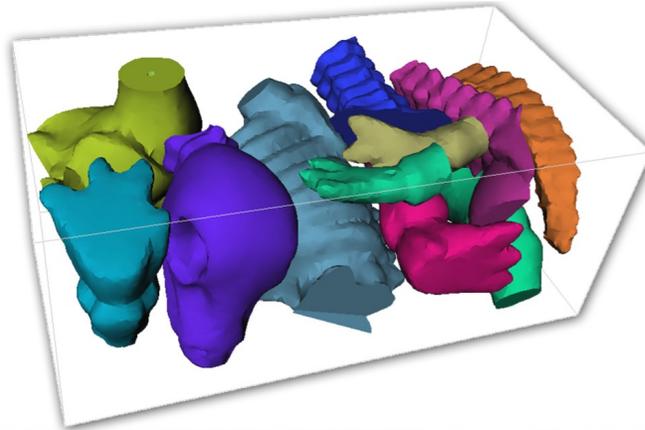
- size
- geometry
- staircase effect
- material consumption
- support artifacts
- material/color
- assemblability
- **collision avoidance**



[Dai et al., SIGGRAPH 2018]

Shape Decomposition for Manufacturing

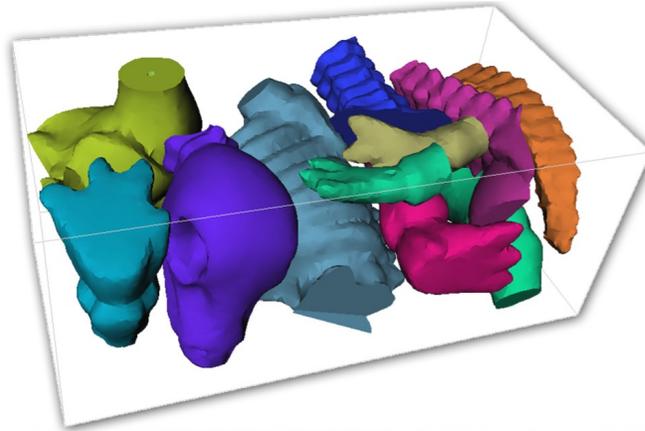
- size
- geometry
- staircase effect
- material consumption
- support artifacts
- material/color
- assemblability
- collision avoidance
- **packing**



[Attene, EG 2015]

Shape Decomposition for Manufacturing

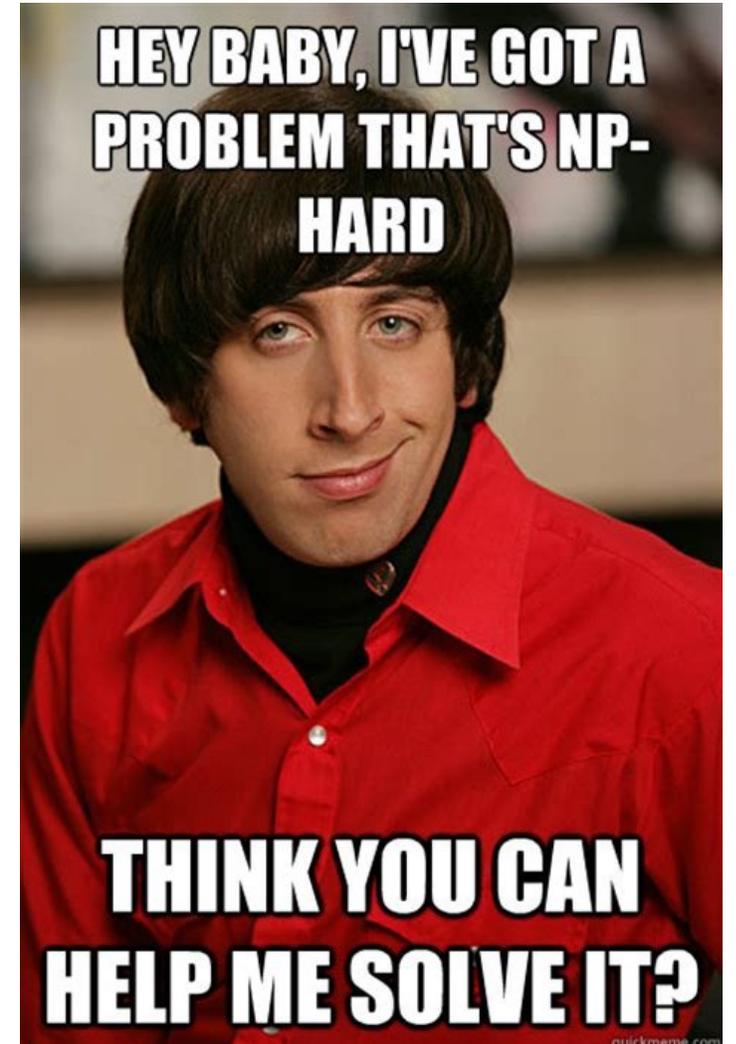
- size
- geometry
- staircase effect
- material consumption
- support artifacts
- material/color
- assemblability
- collision avoidance
- packing
- ... **and many others!**



[Attene, EG 2015]

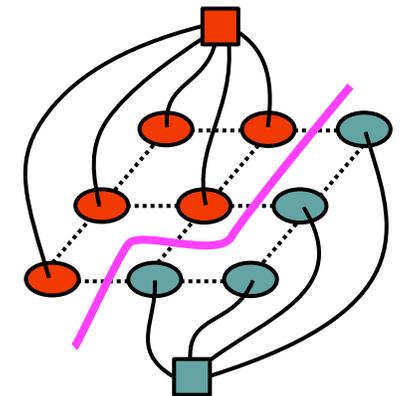
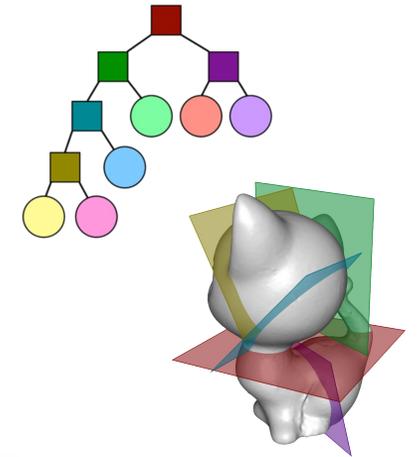
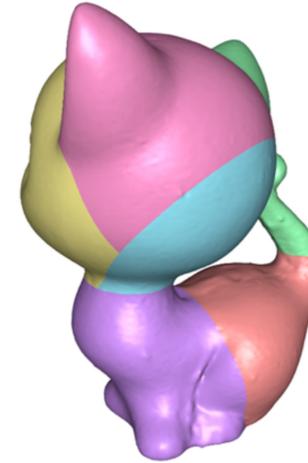
How Hard it is?

- Computing a **decomposition** is **easy**
- Computing a **constrained decomposition** is **still easy**
 - why? I can split into many tiny pieces!
- Computing a **constrained decomposition** that is minimal is **NP-Hard**
 - search space grows exponentially
 - no hope to find the global optimum
 - just strive for a "good" local minimum
 - it's all about heuristics



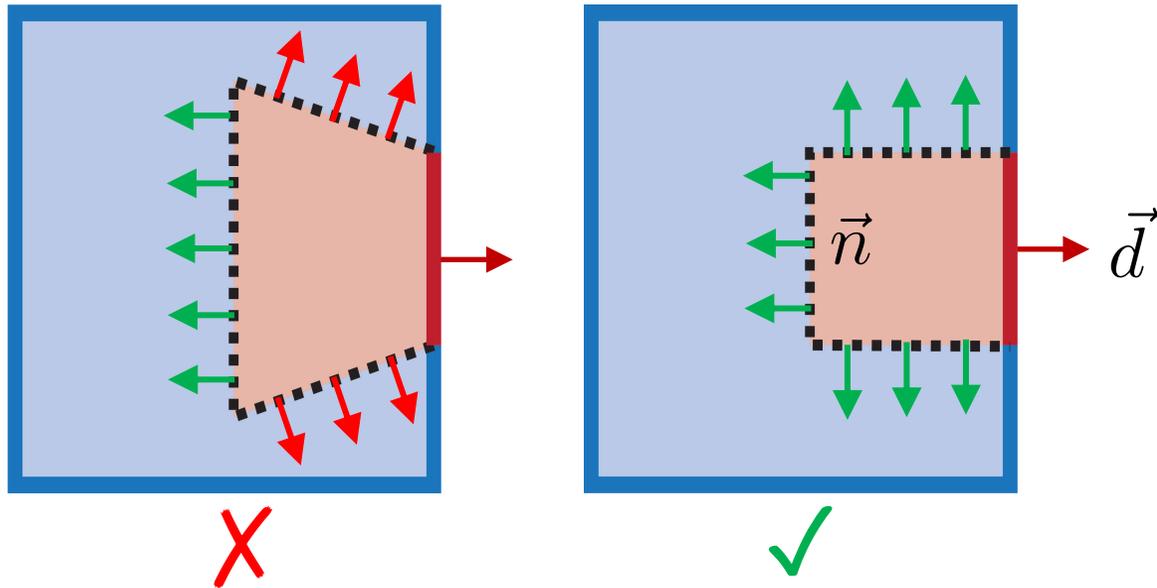
Recurring Approaches

- Despite this variety of goals, manufacturing paradigms and fabrication hardware, all methods
- Aim to control the **same aspects**
 - part size
 - local surface orientation
 - assemblability
- Mostly exploit **similar techniques**
 - Binary Space Partitions
 - Graph Labeling
 - Mesh Booleans



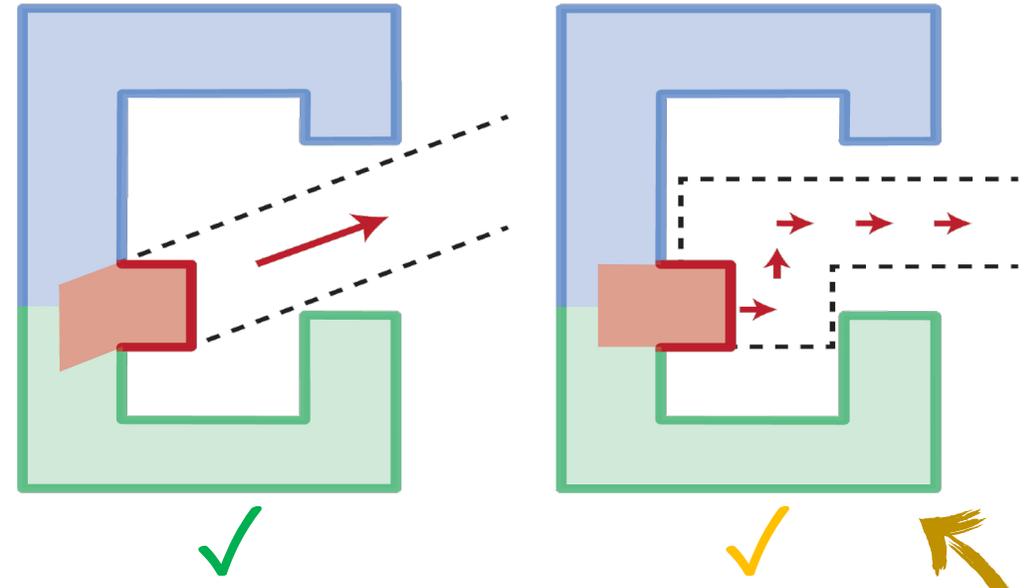
Assemblability – Rigid Bodies

- Parts must fulfill both **local** and **global** requirements



INTERFACES MUST BE **HEIGHT FIELDS**
W.R.T. THE EXTRACTION DIRECTION

$$\vec{d} \cdot \vec{n} \leq 0$$

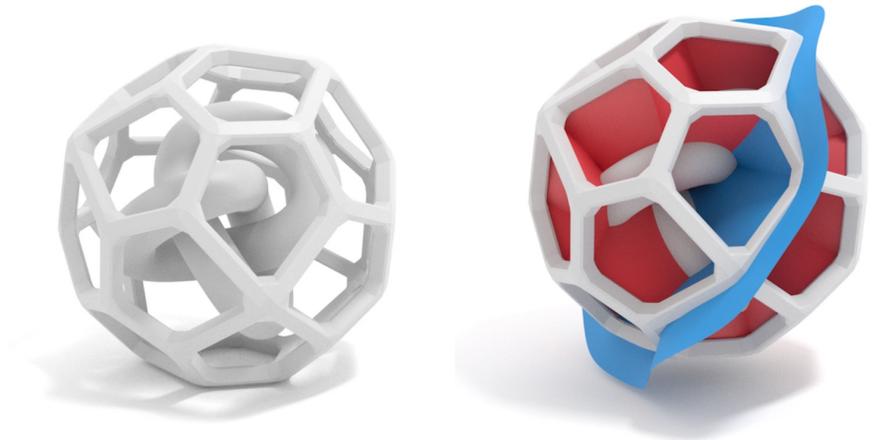


THERE MUST EXIST A **CLEARED**
EXTRACTION PATH

HUGE SEARCH SPACE!

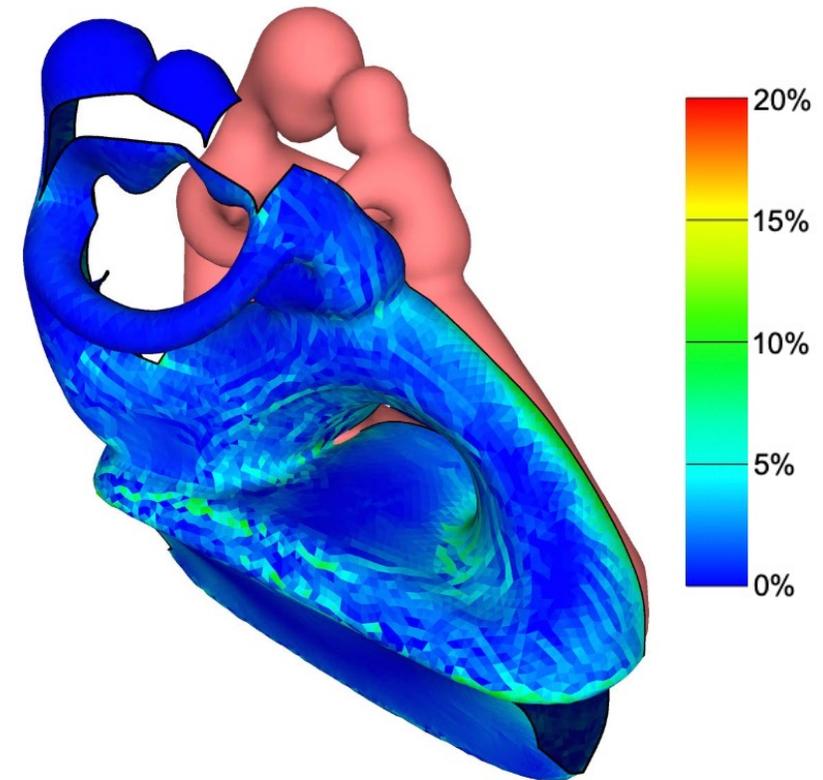
Assemblability – Soft Bodies

- Can be extracted even if they **violate** height fieldness
 - full FEM simulation is overly expensive
 - simulating contact and frictional forces is hard!



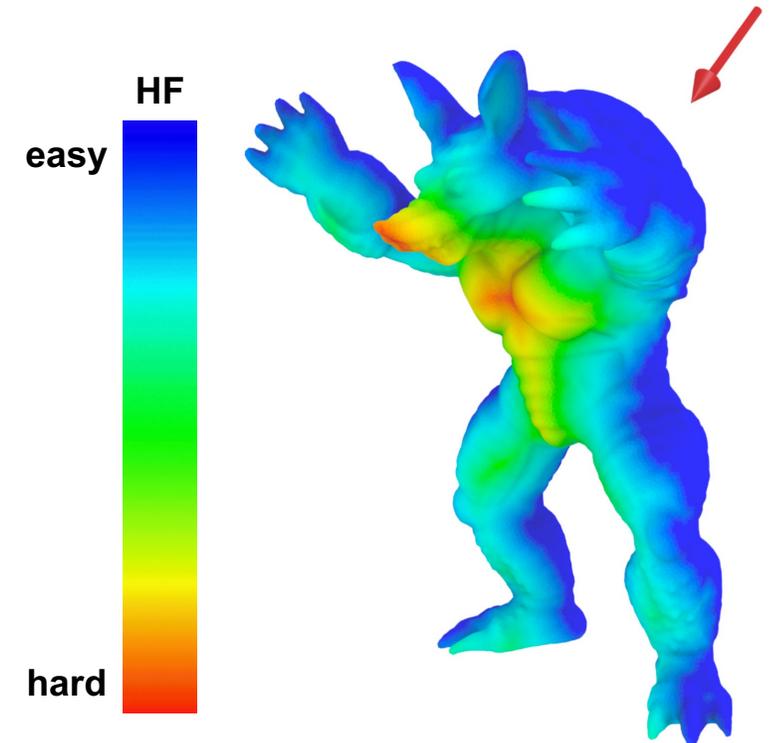
Assemblability of Deformable Bodies

- Can be extracted even if they **violate** height fieldness
 - full FEM simulation is overly expensive
 - simulating contact and frictional forces is hard!
- **FlexMolds approach**
 - projective dynamics on a thin shell
 - measure per triangle deformation during extraction



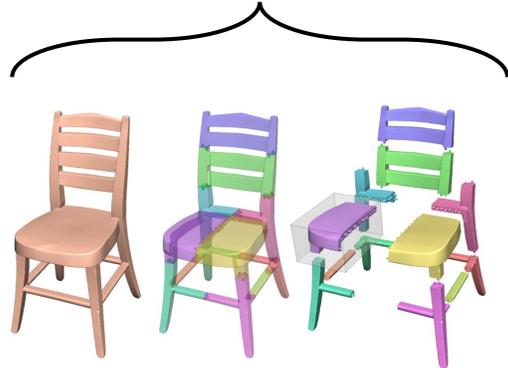
Assemblability of Deformable Bodies

- Can be extracted even if they **violate** height fieldness
 - full FEM simulation is overly expensive
 - simulating contact and frictional forces is hard!
- **FlexMolds approach**
 - projective dynamics on a thin shell
 - measure per triangle deformation during extraction
- **MetaMolds approach**
 - fully geometric
 - geodesic distance from closest HF region
 - works remarkably well in practice!

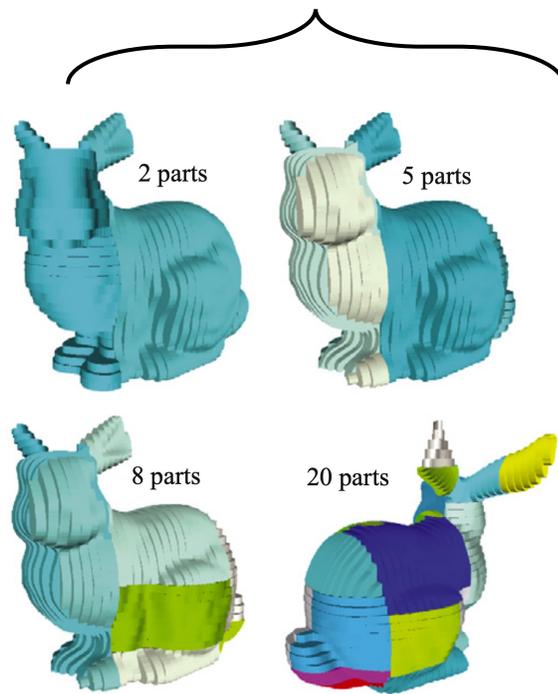


Binary Space Partitions

Size



Staircase Effect



Packing



[Luo et al., SIGGRAPH 2012]

[Hildebrand et al., SMI 2013]

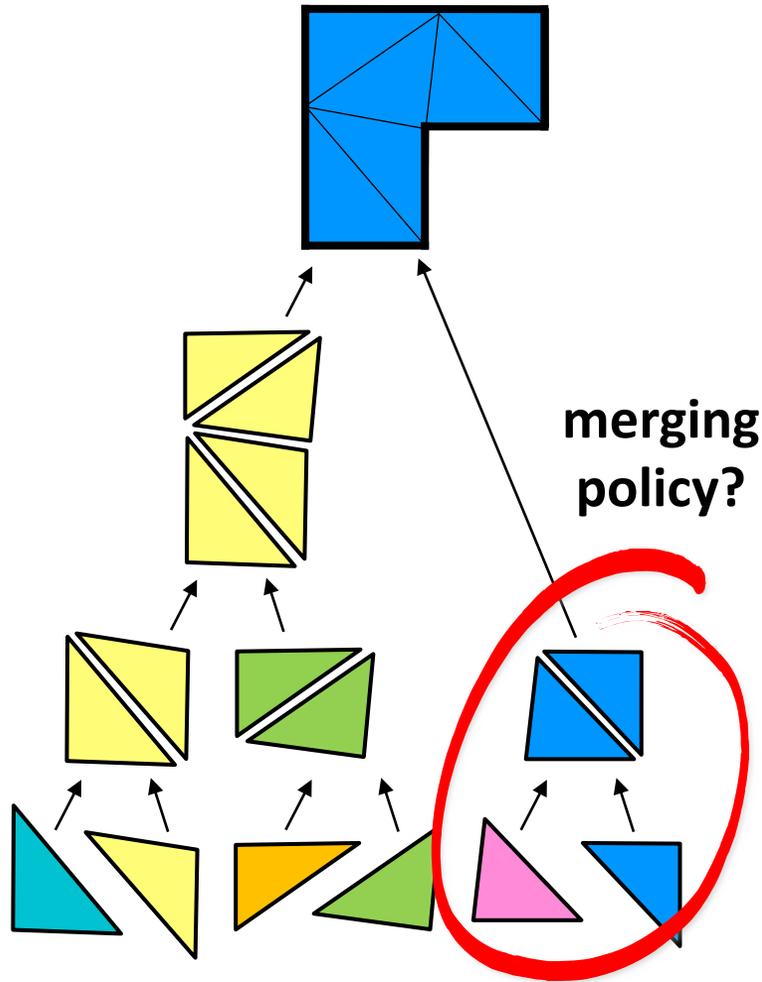
[Chen et al., SIGGRAPH Asia 2015]

[Attene, EG 2015]

Top Down
(with beam search)

Bottom Up
(greedy)

Top Down vs Bottom Up



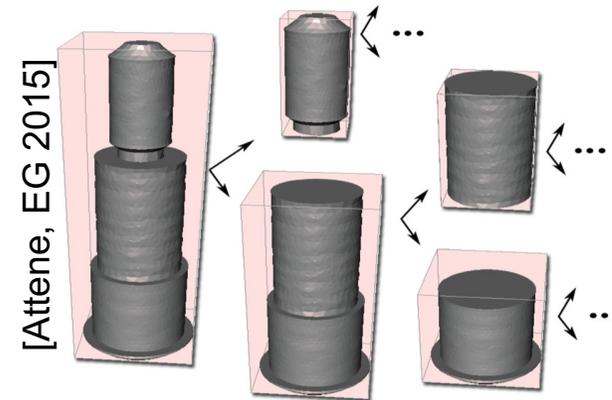
Shapes in a Box:

- **discretize** domain (tetrahedralization)
- minimize absolute aboxiness

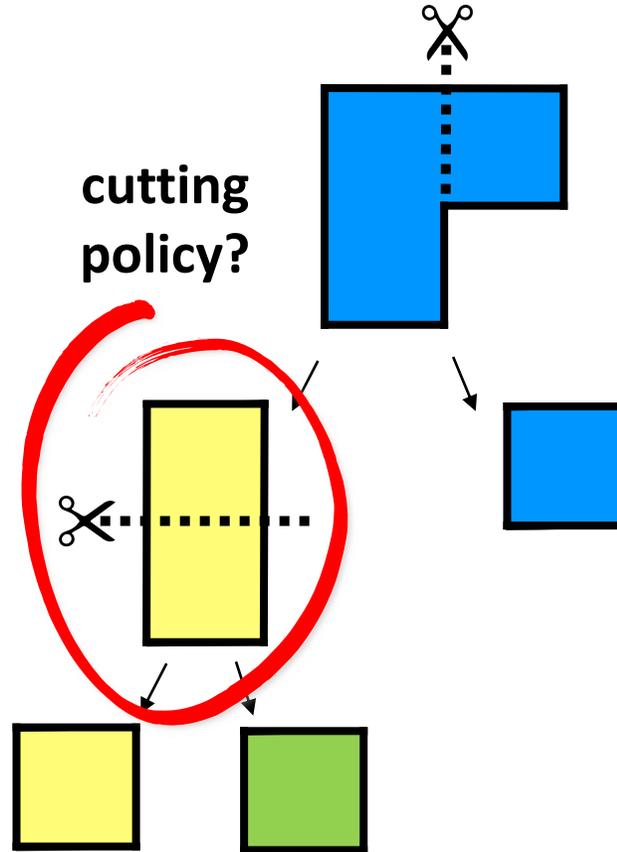
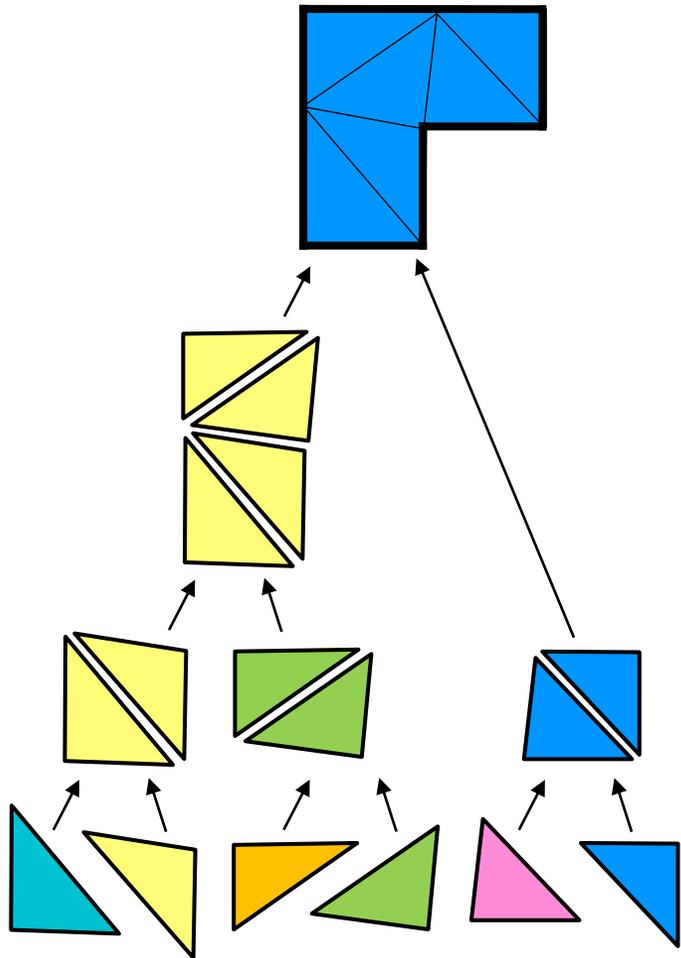
$$A(\mathcal{O}) = VOL(OBB(\mathcal{O})) - VOL(\mathcal{O})$$

↑
cluster
of
tetrahedra

↑
minimal
oriented
bbox

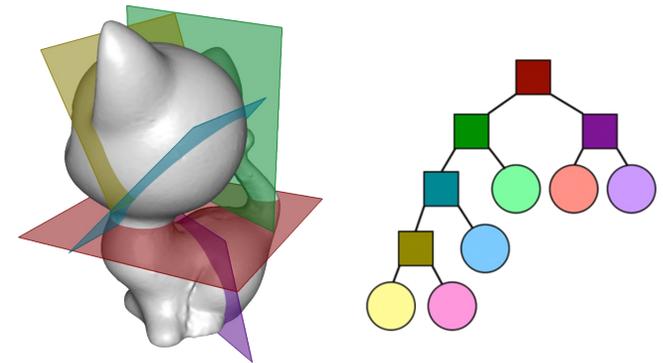


Top Down vs Bottom Up



Chopper:

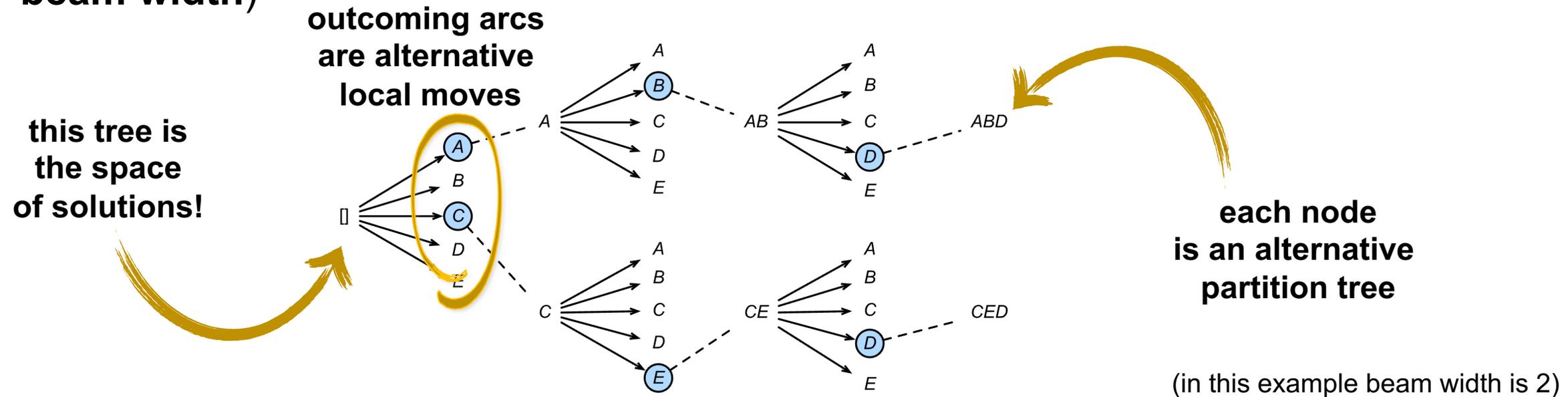
- **discretize** set of planar cuts
- minimize split metric based on
 - # of parts
 - connectors
 - structure/fragility
 - aesthetics (hide seams)
 - symmetry



[Luo et al., SIGGRAPH 2012]

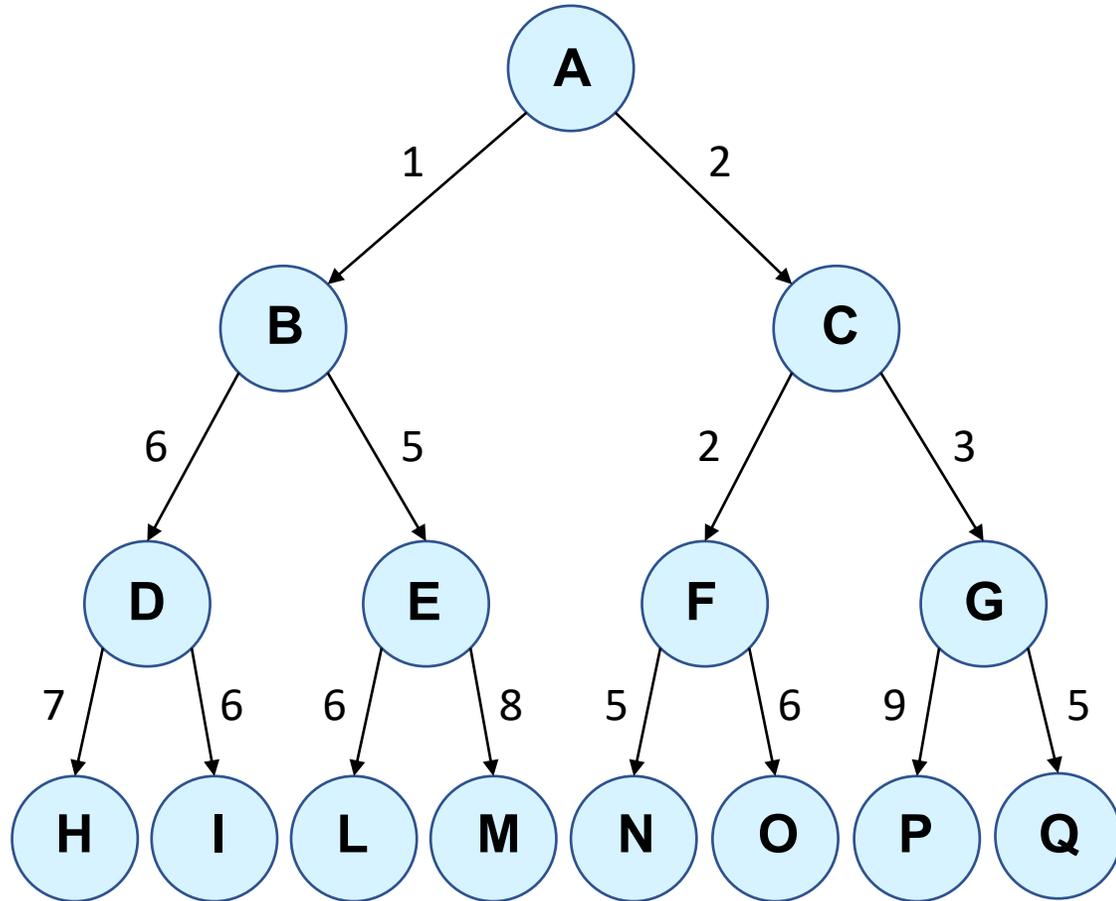
Exploring the Space of Solutions

- **Greedy:** at each step pick the best move
 - easy to get stuck at local minima!
- **Beam search** allows to explore a wider portion of the feasible space
 - assumption: partial solutions can be ranked
 - idea: at each level, continue exploring only the N best partial solutions (N is called **beam width**)

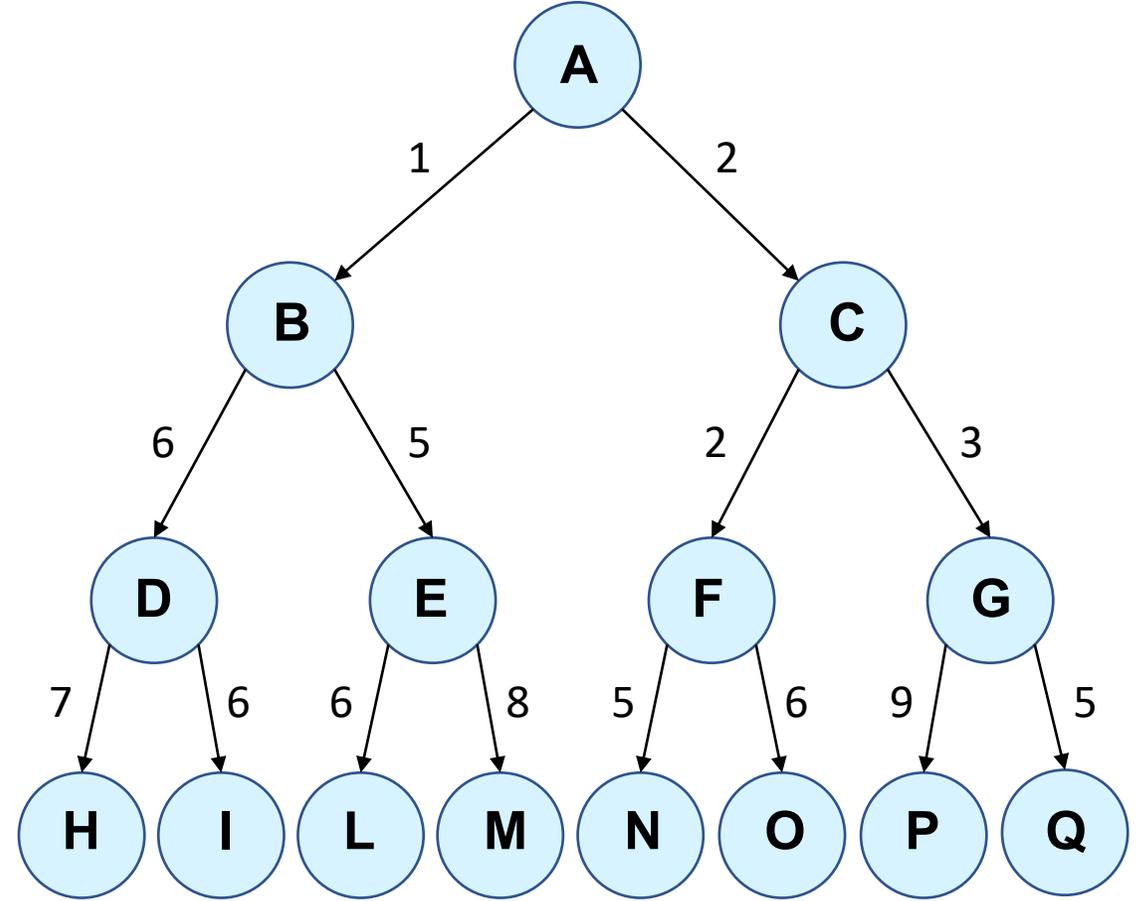


Beam Search vs Greedy

Beam Search, N=2

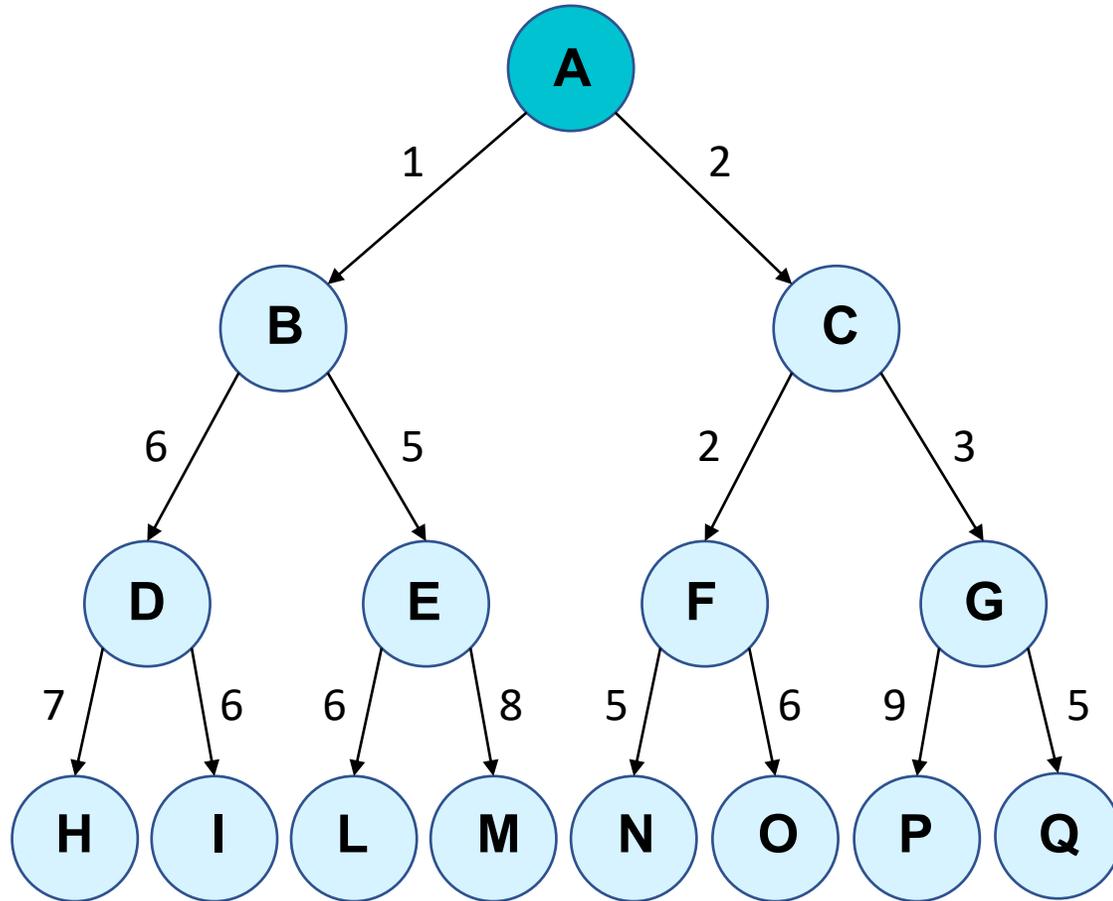


Greedy

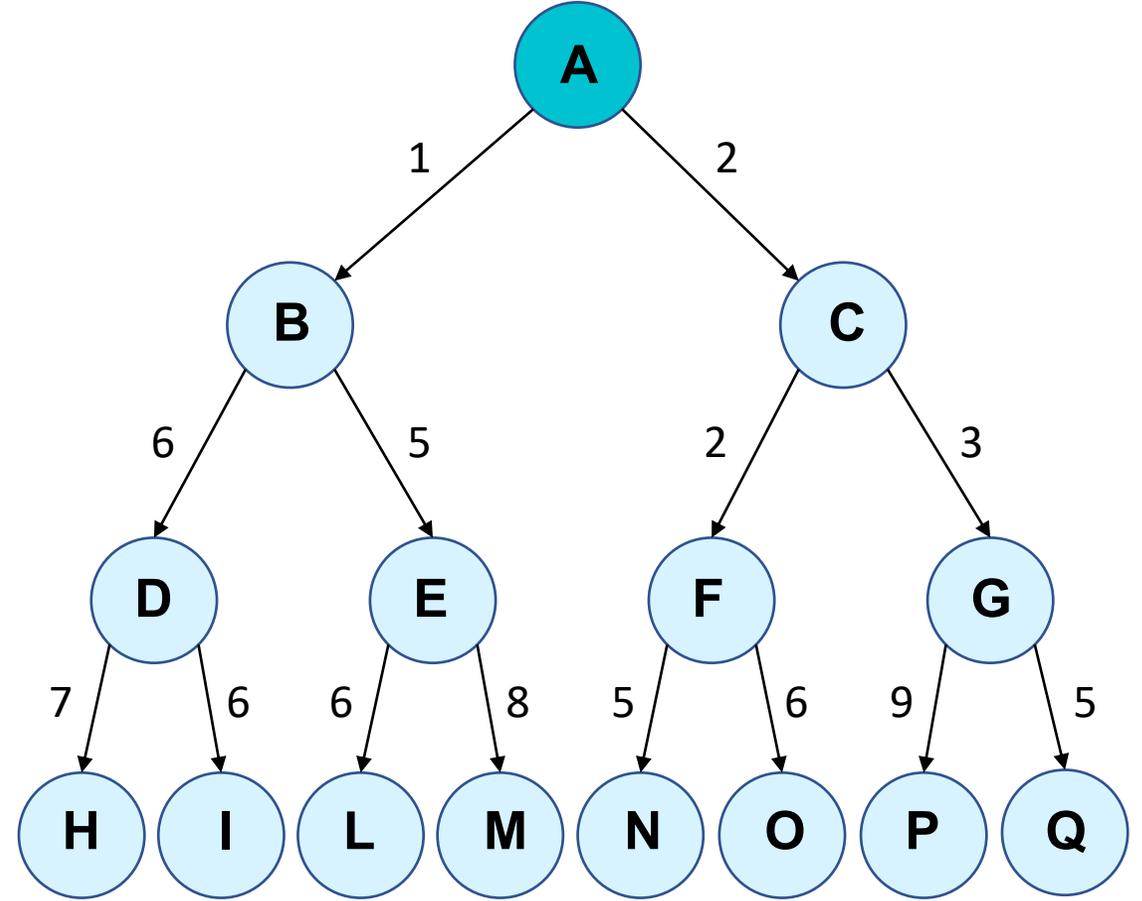


Beam Search vs Greedy

Beam Search, N=2

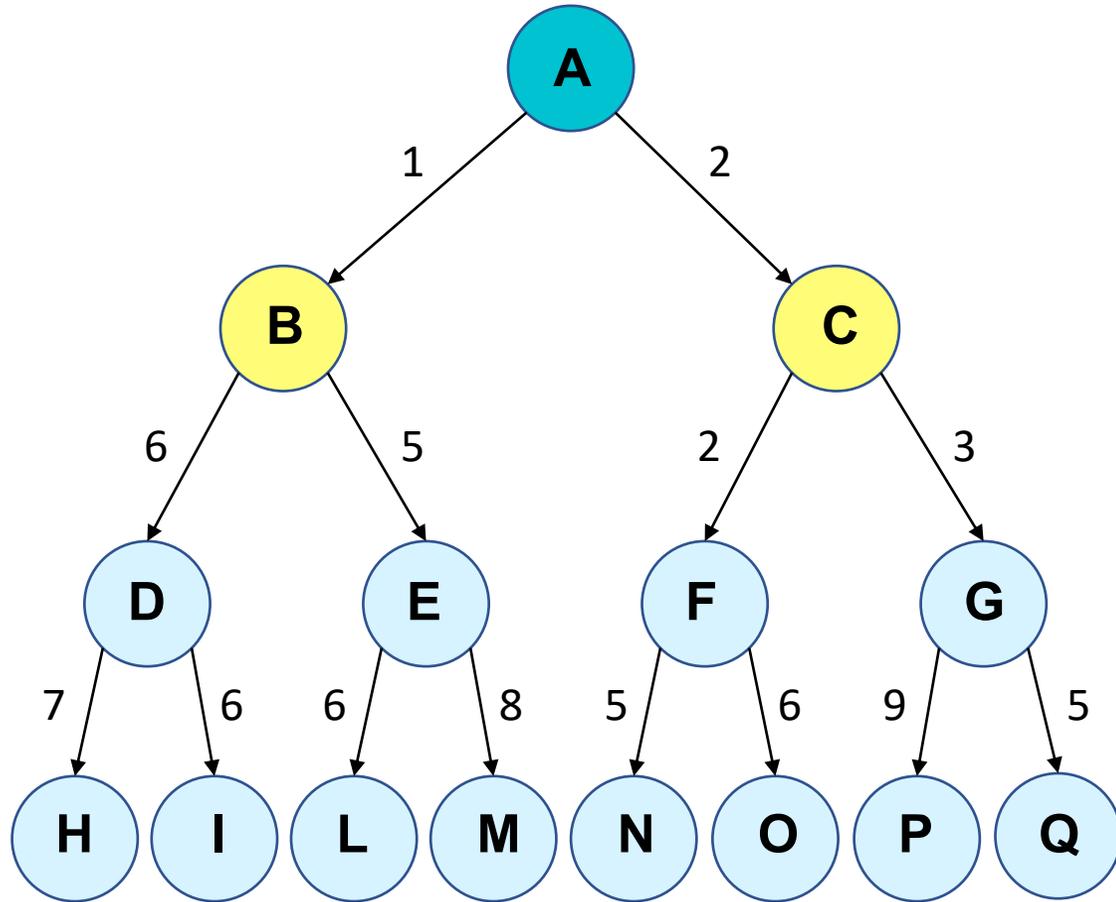


Greedy

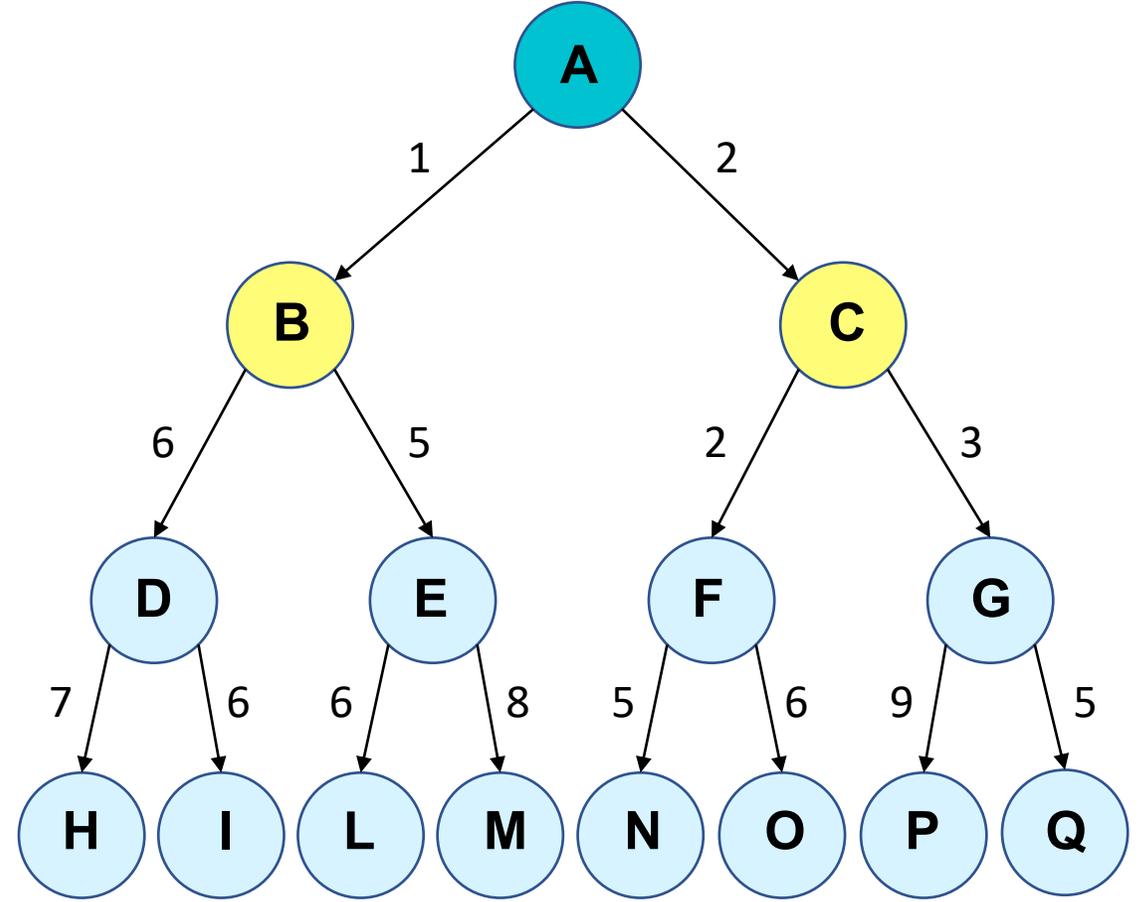


Beam Search vs Greedy

Beam Search, N=2

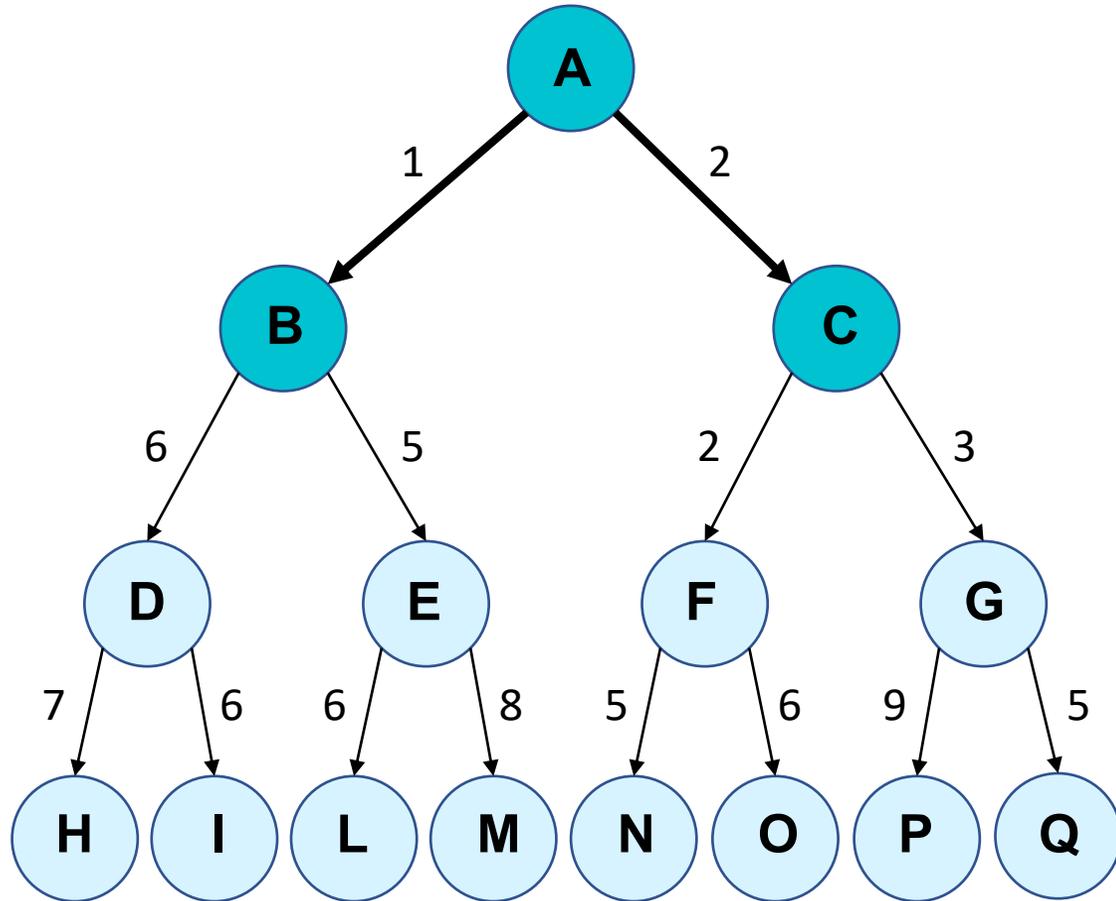


Greedy

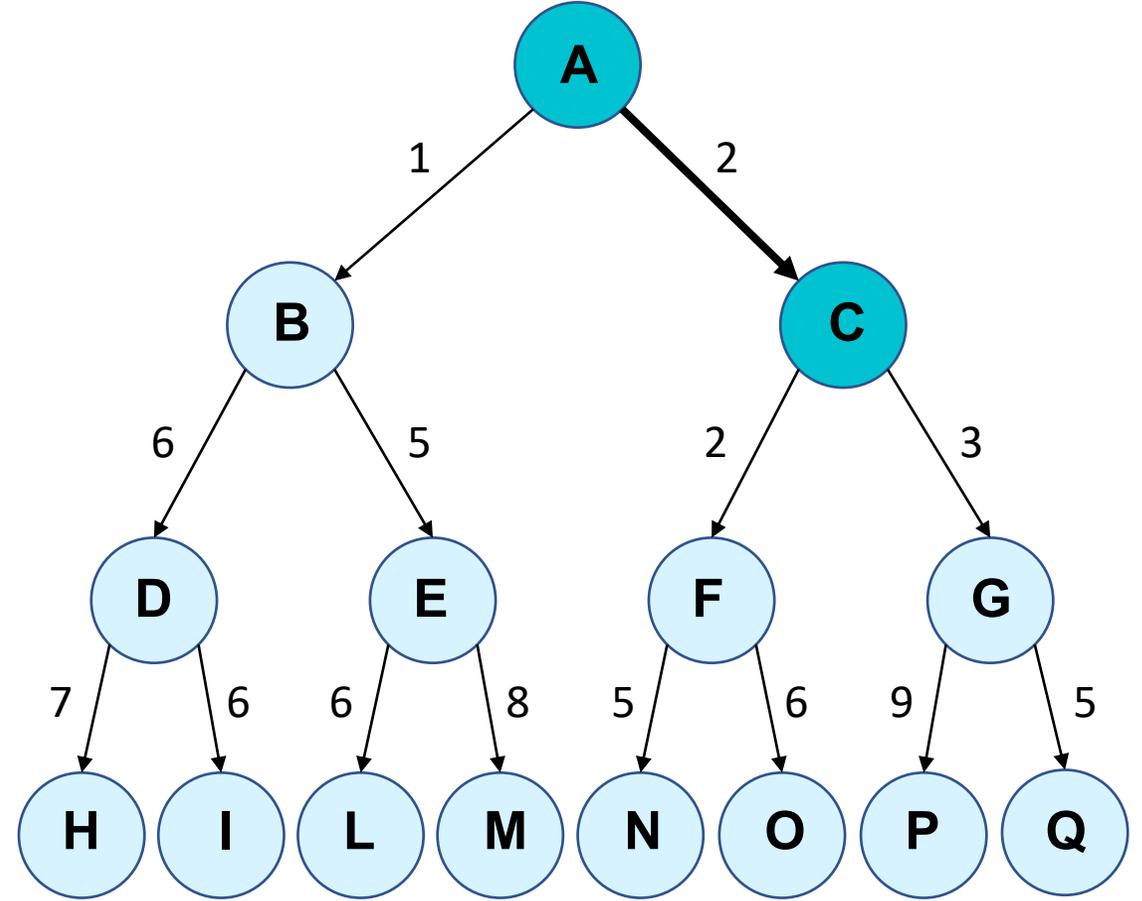


Beam Search vs Greedy

Beam Search, N=2

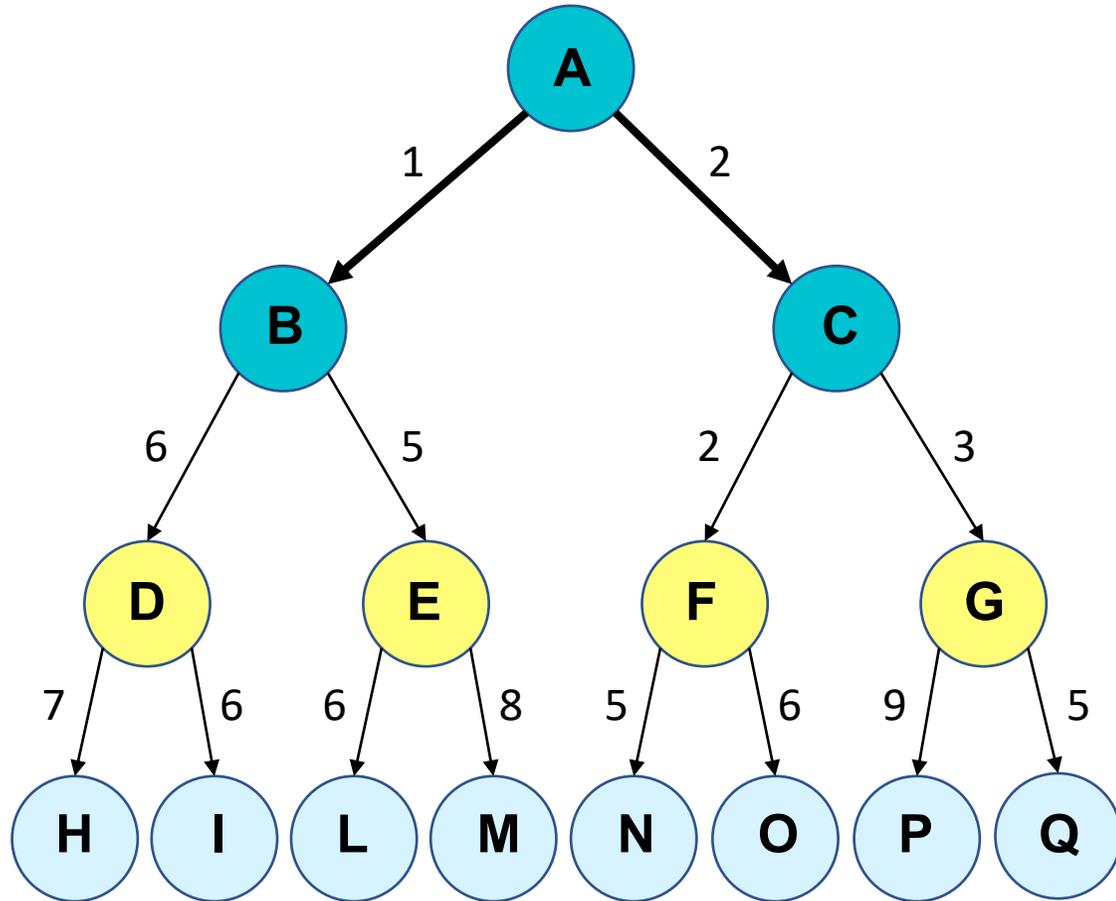


Greedy

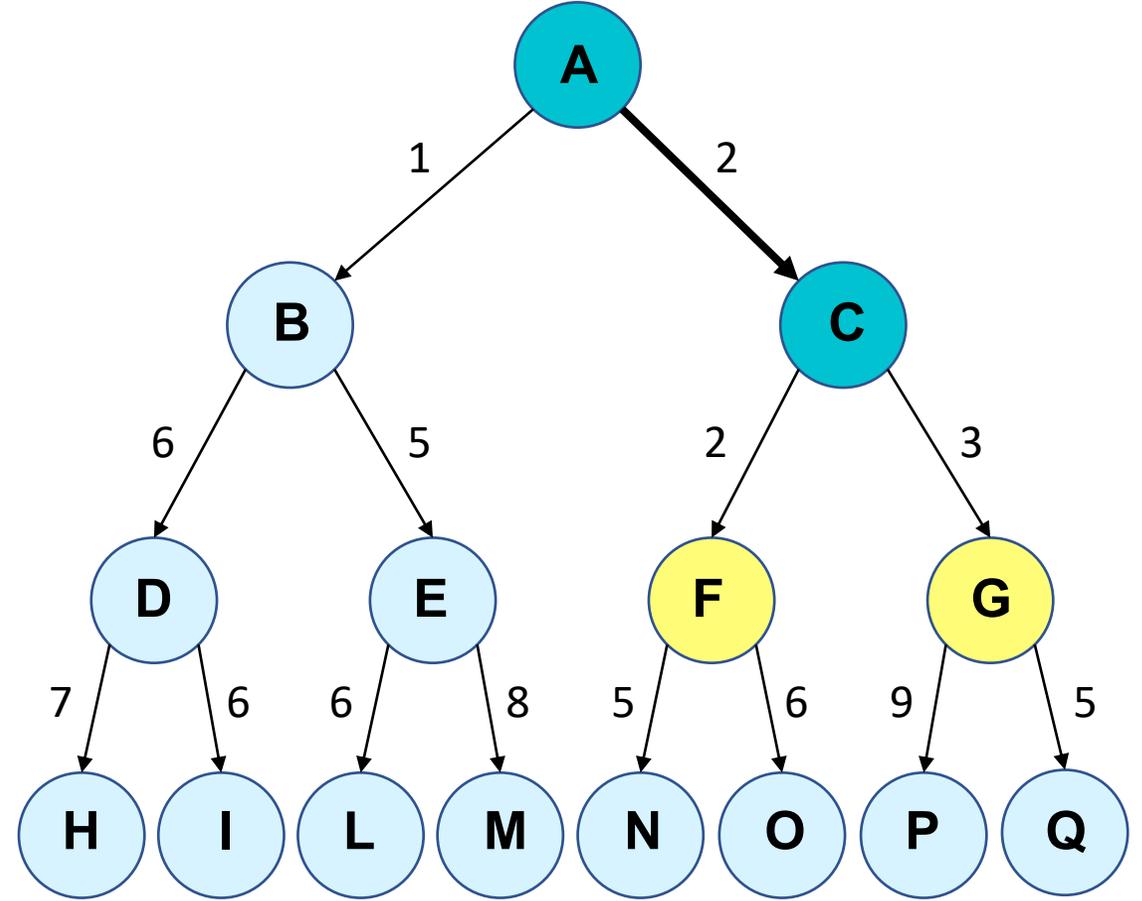


Beam Search vs Greedy

Beam Search, N=2

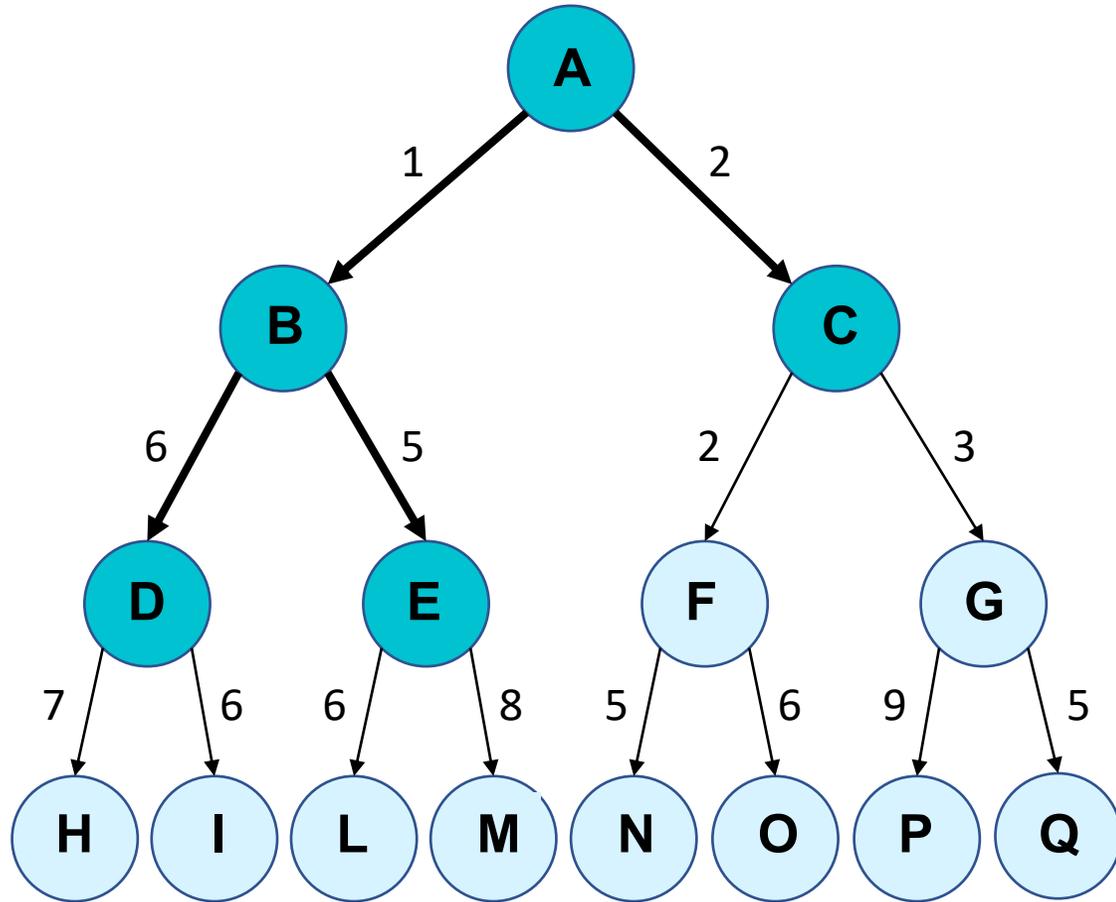


Greedy

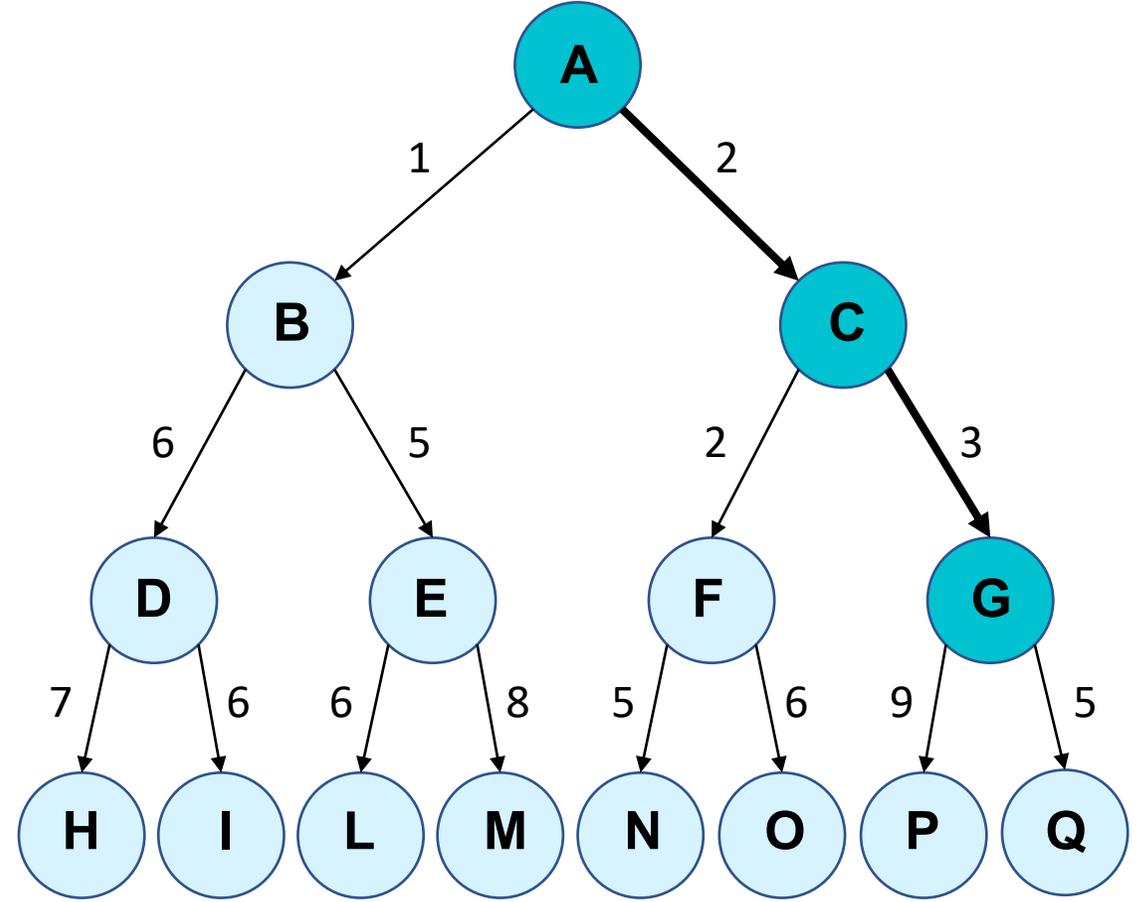


Beam Search vs Greedy

Beam Search, N=2

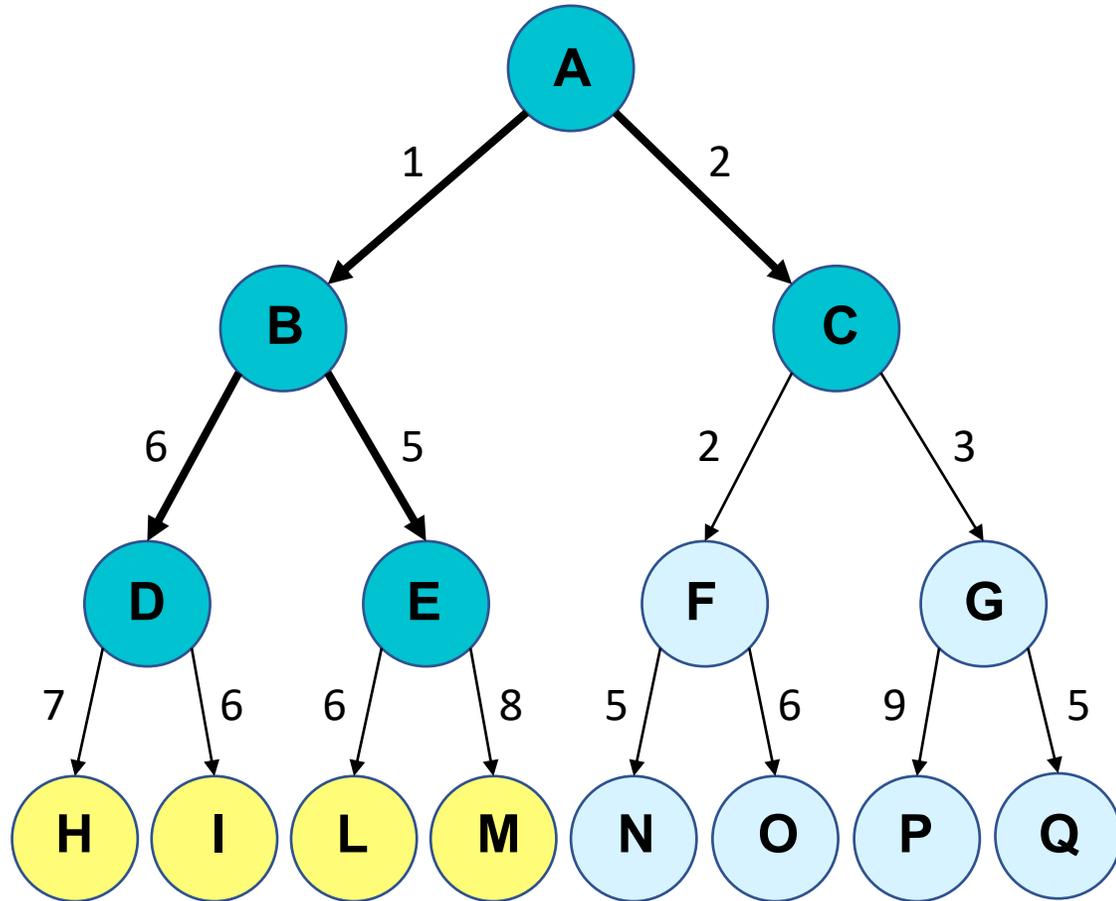


Greedy

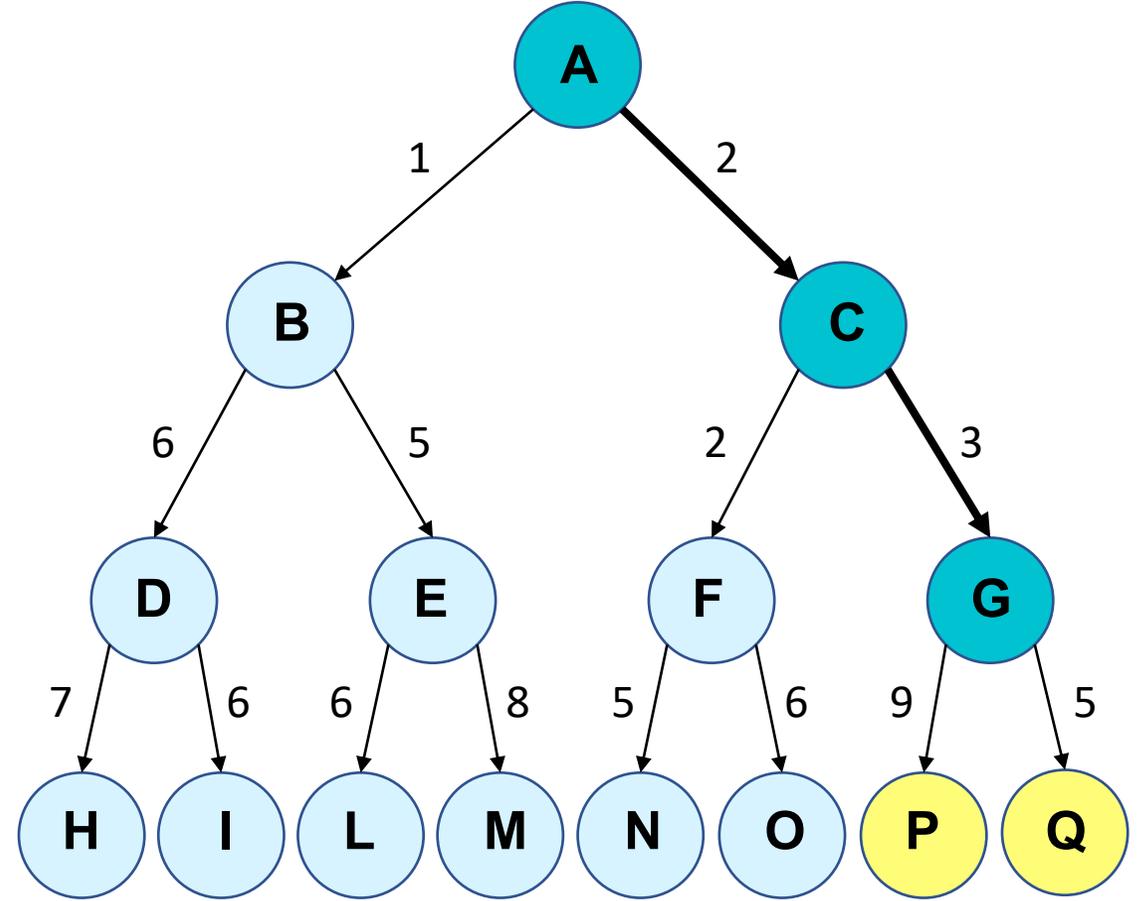


Beam Search vs Greedy

Beam Search, N=2

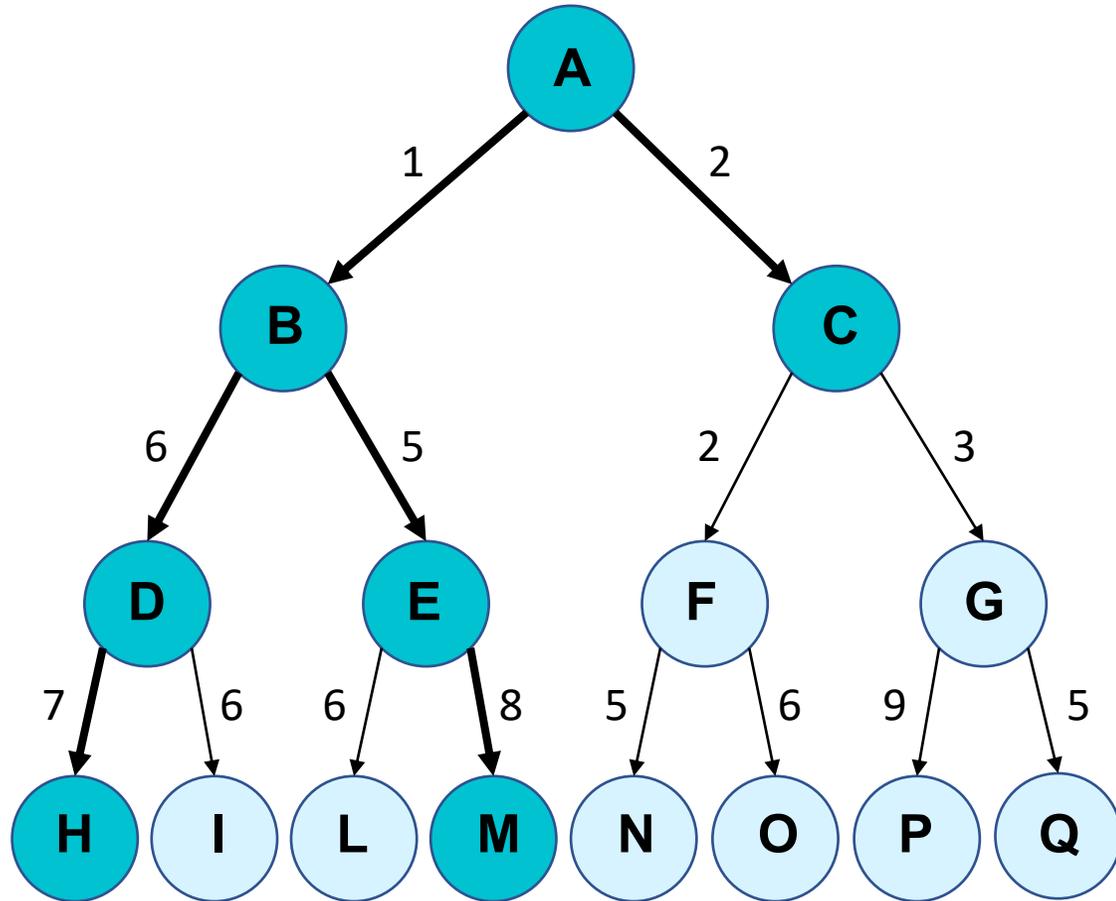


Greedy

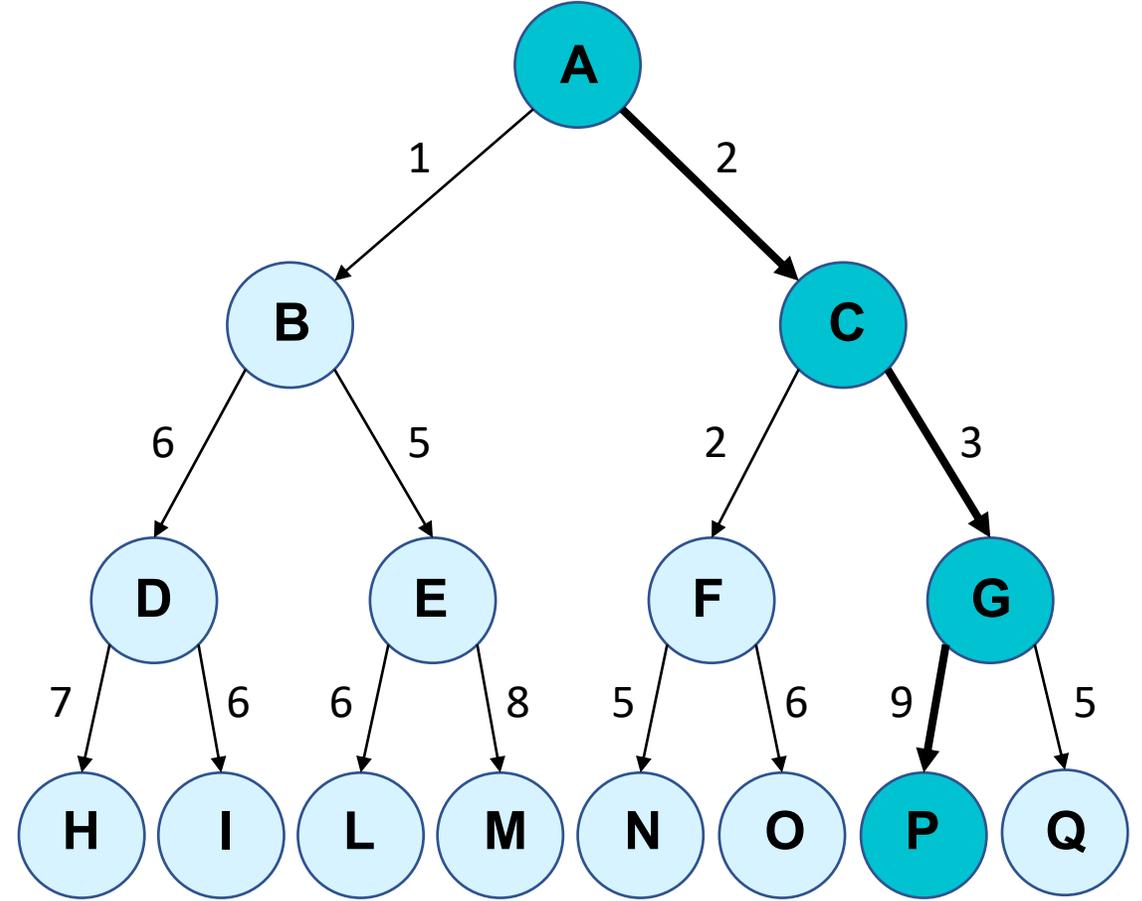


Beam Search vs Greedy

Beam Search, N=2

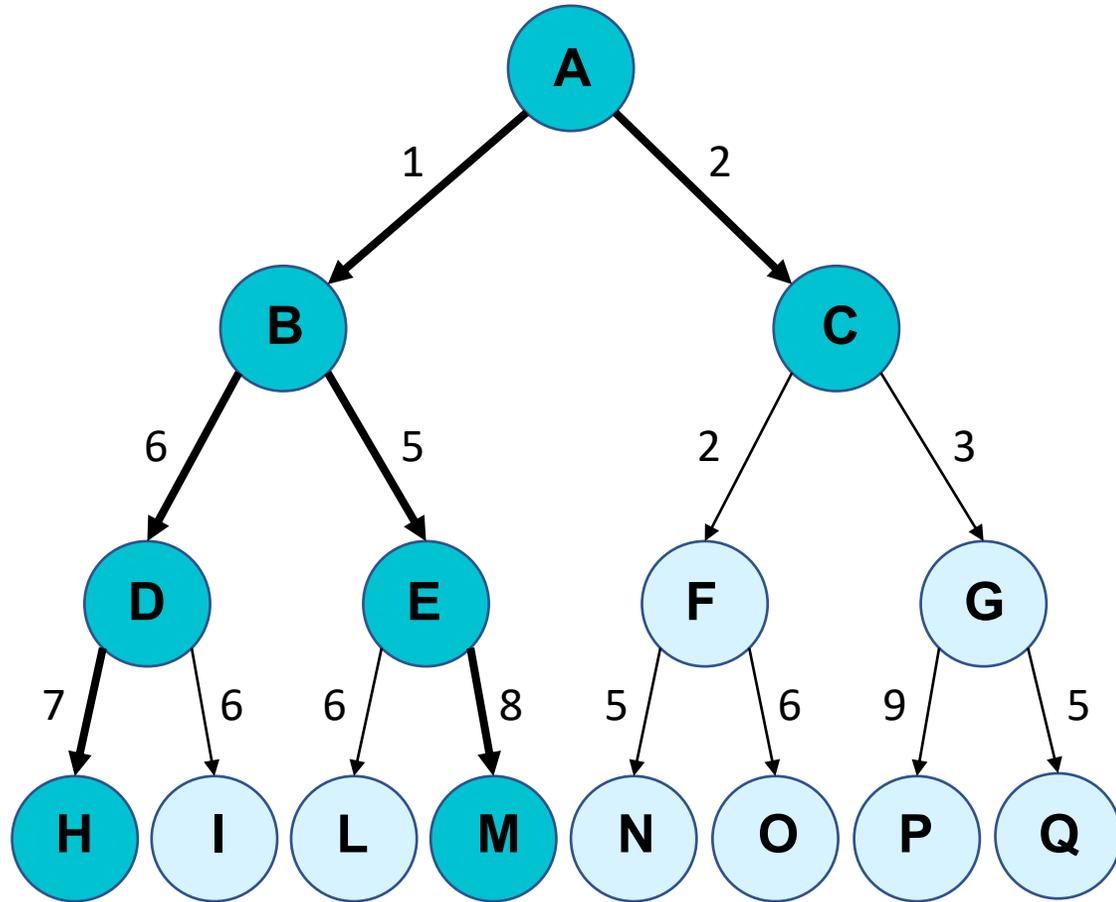


Greedy

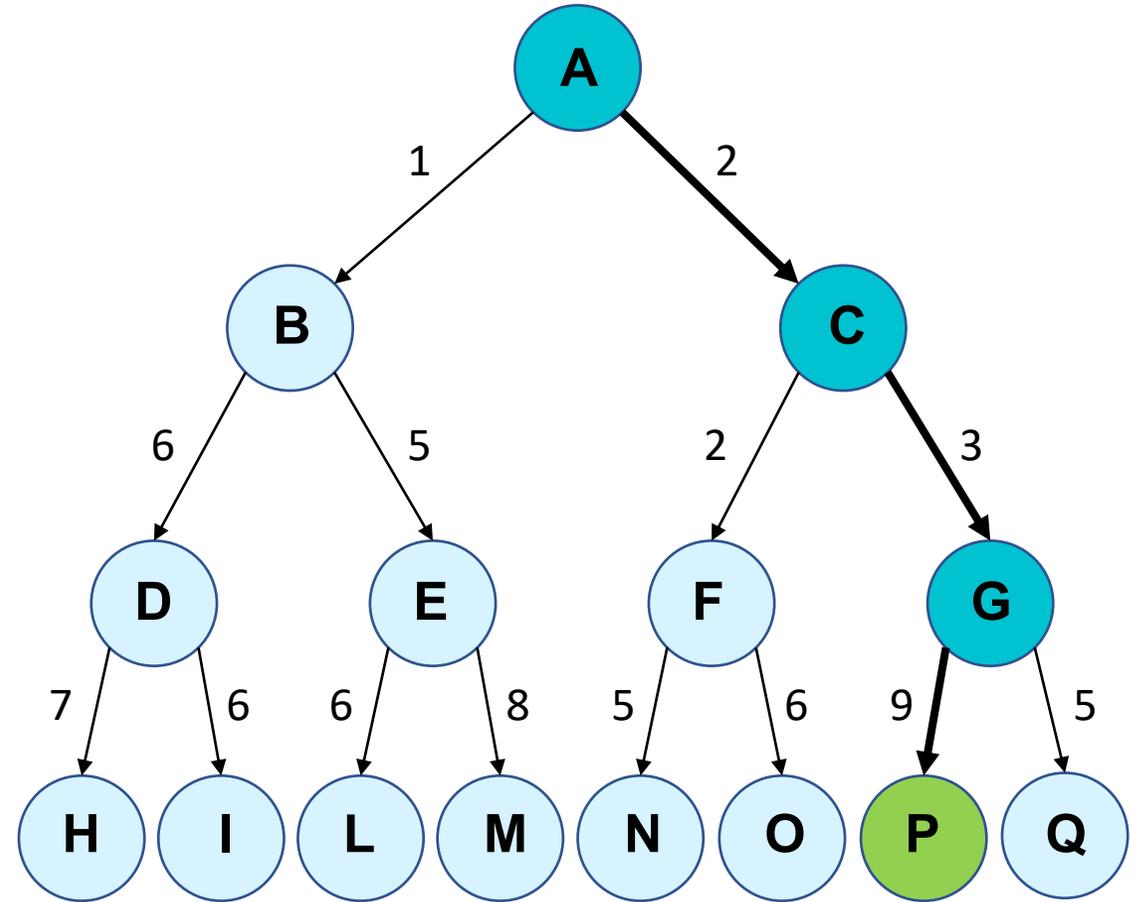


Beam Search vs Greedy

Beam Search, N=2

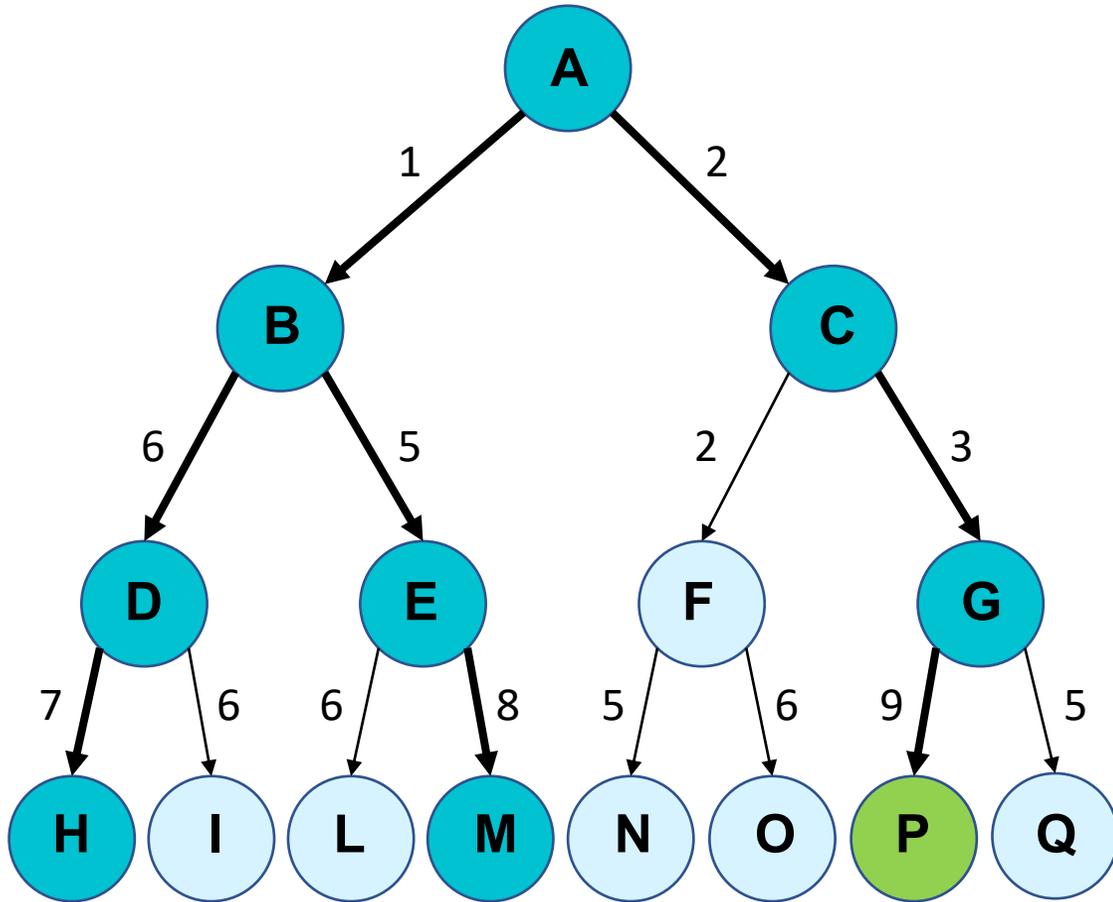


Greedy

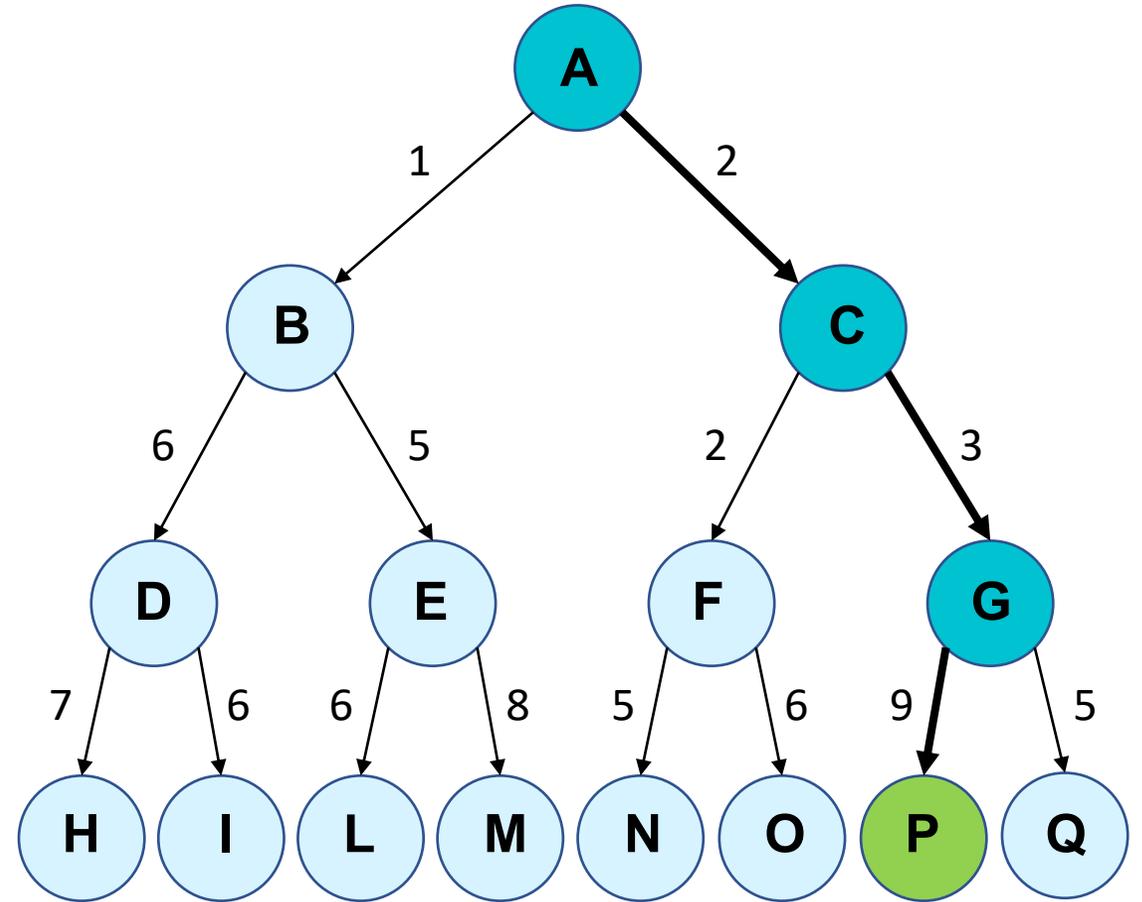


Beam Search vs Greedy

Beam Search, **N=3**



Greedy



Labeling



[Herholz et al., EG 2015]

ILP, GraphCut



[Alderighi et al., SIGGRAPH 2019]

Greedy



[Filoscia et al., EG 2020]

Over segmentation +
region merging

Graph Cuts

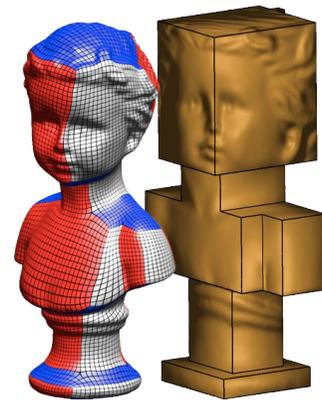
- Solves a multi-labeling problem on a generic graph $G(N,A)$ by minimizing

$$L = \arg \min \sum_{i \in N} C_i(l) + \sum_{ij \in A} C_{ij}(l_i, l_j)$$

DATA TERM
cost of assigning
label l to node i

SMOOTH TERM
cost of assigning
labels l_i, l_j to
adjacent nodes i, j

- The problem is NP-Complete
 - finds a local minimum
 - depends on initialization and processing order
 - heavily used in Vision/Graphics
 - it works remarkably well in practice!



[PolyCut, SIG Asia 2013]

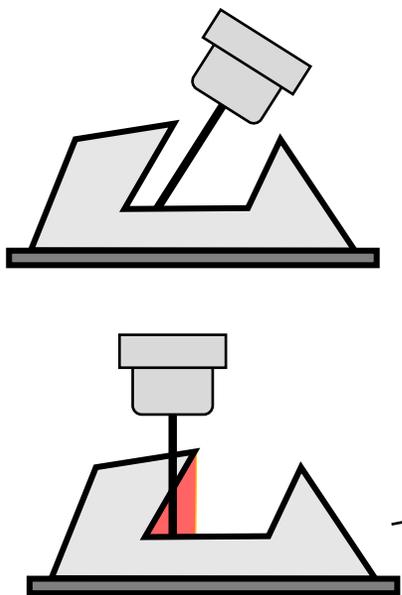


[GrabCut, SIGGRAPH 2004]

Graph Cuts for Digital Manufacturing

- The **graph** is the dual mesh
 - one node per triangle / tetrahedron / voxel
- The **labels** are candidate machining / extraction directions

$$L = \arg \min \sum_{i \in N} C_i(l) + \sum_{ij \in A} C_{ij}(l_i, l_j)$$



DATA TERM

$\left\{ \begin{array}{ll} f_i(l) & \text{if feasible} \\ \infty & \text{otherwise} \end{array} \right.$

SMOOTH TERM

$\left\{ \begin{array}{ll} 0 & \text{if } l_i = l_j \\ f_{ij}(l_i, l_j) & \text{otherwise} \end{array} \right.$

Graph Cuts in Digital Manufacturing

- The **graph** is the dual mesh
 - one node per triangle / tetrahedron / voxel
- The **labels** are candidate machining / extraction directions

Surface2Volume

G: dual tetmesh
L: extraction directions



[Araùjo et al., SIGGRAPH 2019]

HF Decomp

G: dual trimesh
L: HF directions



[Herholz et al., EG 2015]

4 Axis Milling

G: dual trimesh
L: milling directions



[Nuvoli et al., EG 2021]

Rigid Molding

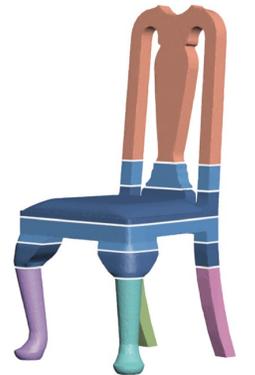
G: dual tetmesh
L: molding directions



[Alderighi et al., SIG Asia 2021]

DHF Slicer

G: dual trimesh
L: DHF directions

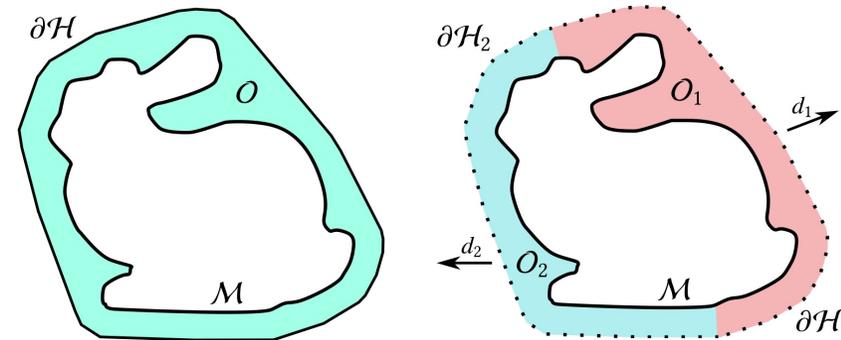


[Yang et al., SIG Asia 2020]

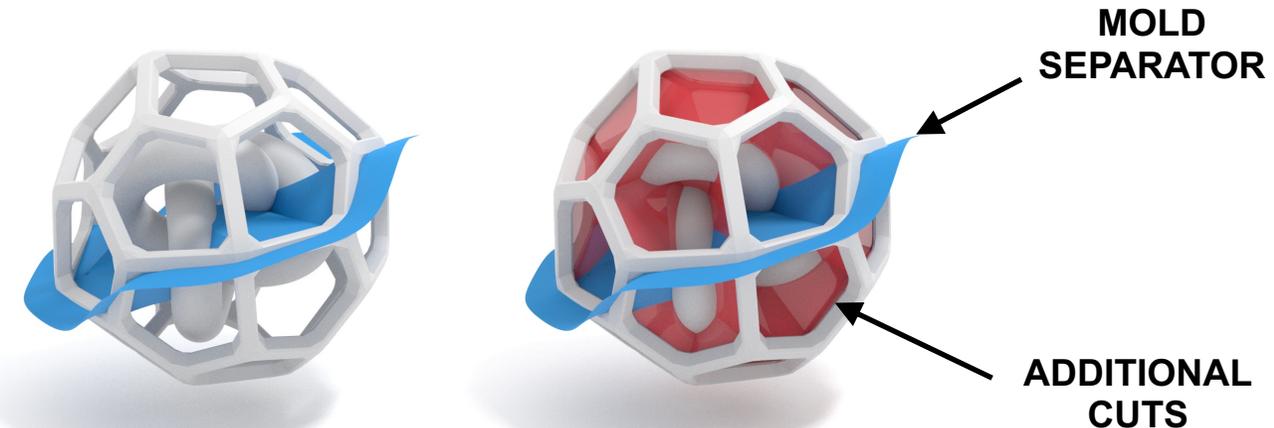
Greedy Labeling for Volume-Aware molding

- Greedy labeling does not typically work as well as graph cuts, but when you have the **right idea**, it becomes extremely powerful!

- bipartition the **outer shell** finding two mold directions that maximize visual coverage



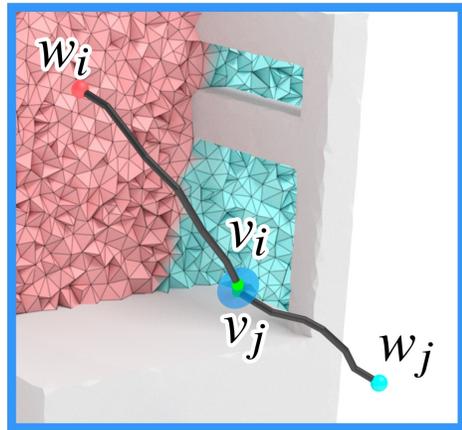
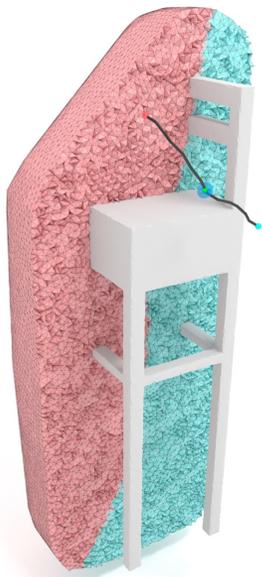
- propagate the labeling **inside the volume**, also defining additional cuts



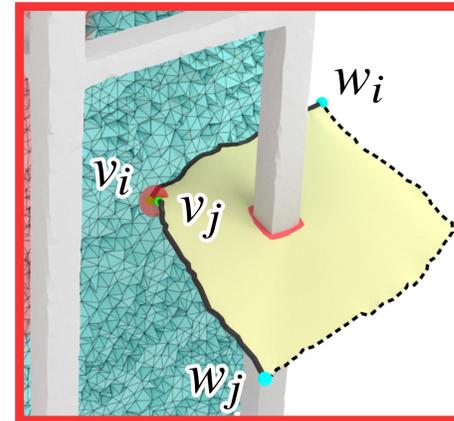
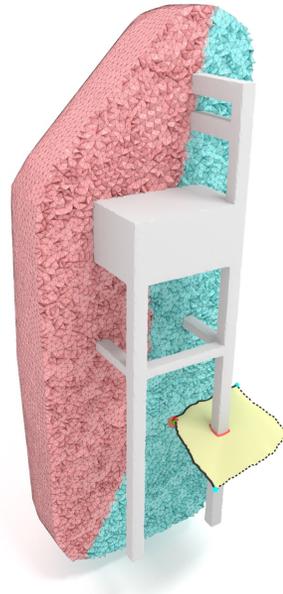
[Alderighi et al., SIGGRAPH 2019]

Greedy Labeling for Volume-Aware molding

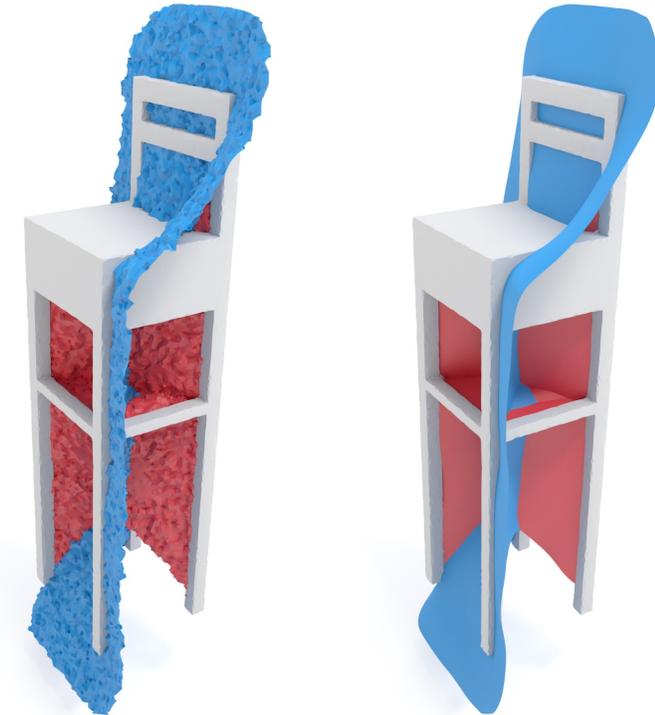
- Greedy labeling does not typically work as well as graph cuts, but when you have the **right idea**, it becomes extremely powerful!



LOCAL CONDITION
FOR MOLD
SEPARATOR



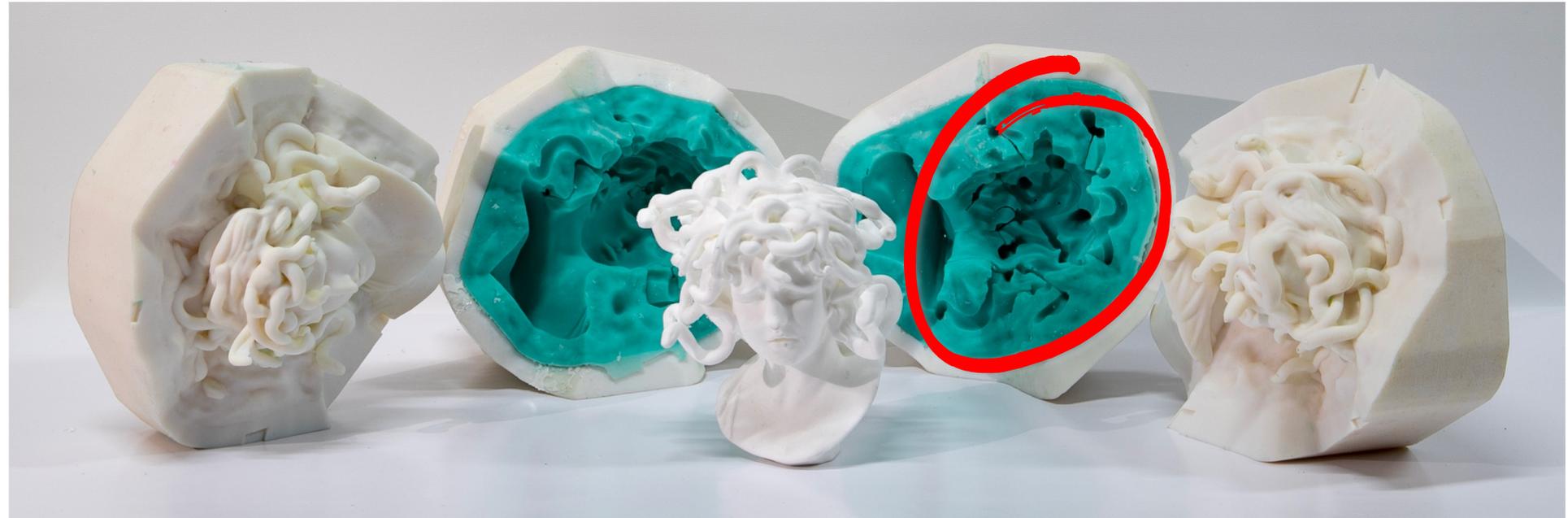
LOCAL CONDITION
FOR CUTTING
MEMBRANE



[Alderighi et al., SIGGRAPH 2019]

Greedy Labeling for Volume-Aware molding

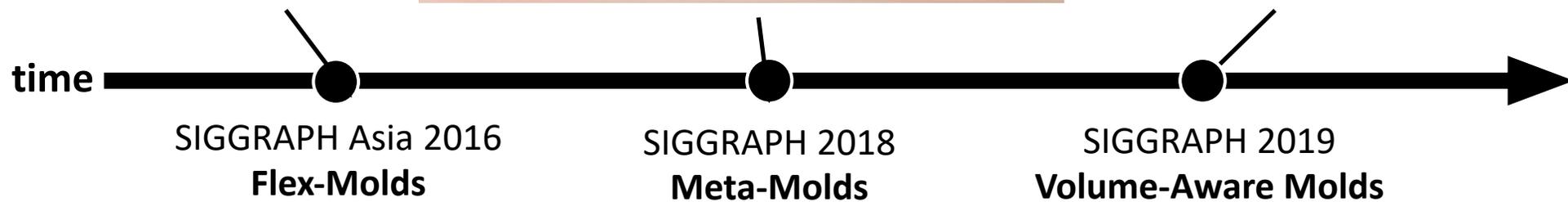
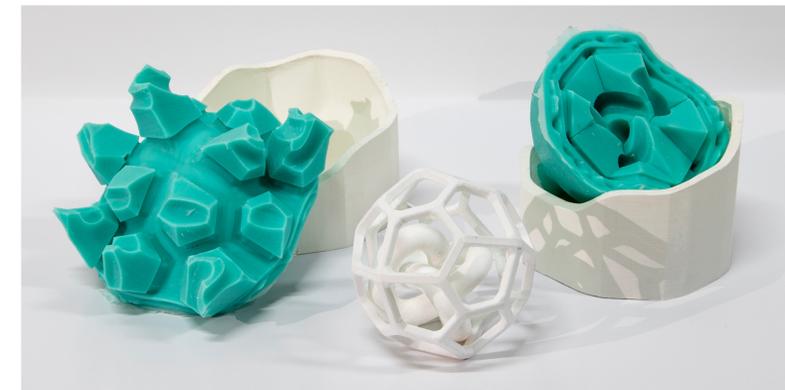
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[Alderighi et al., SIGGRAPH 2019]

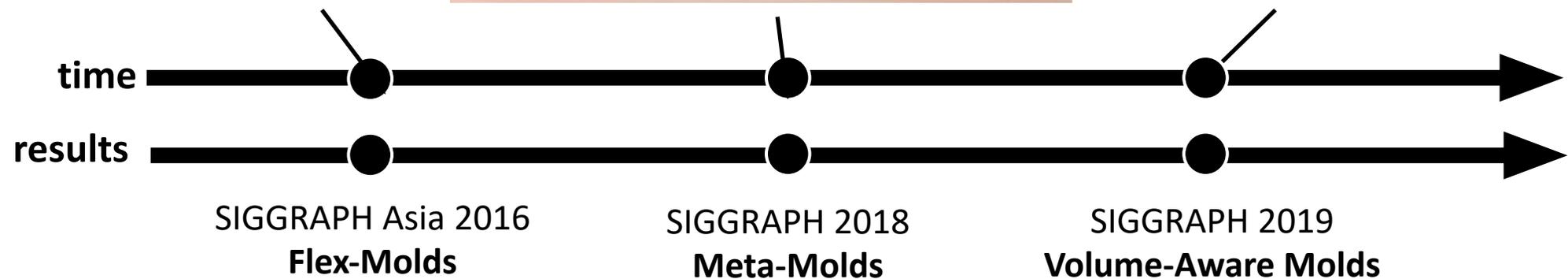
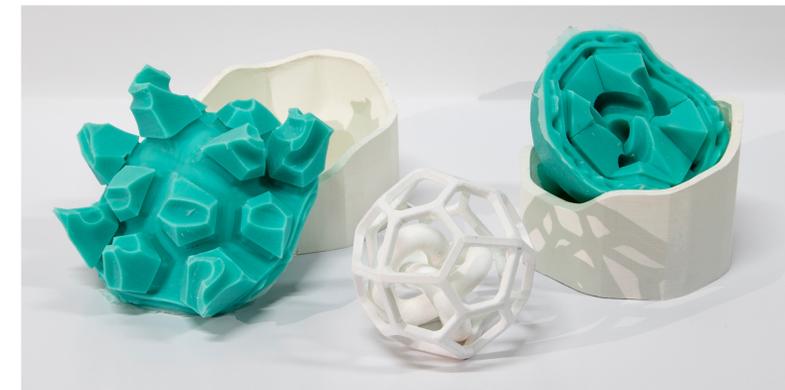
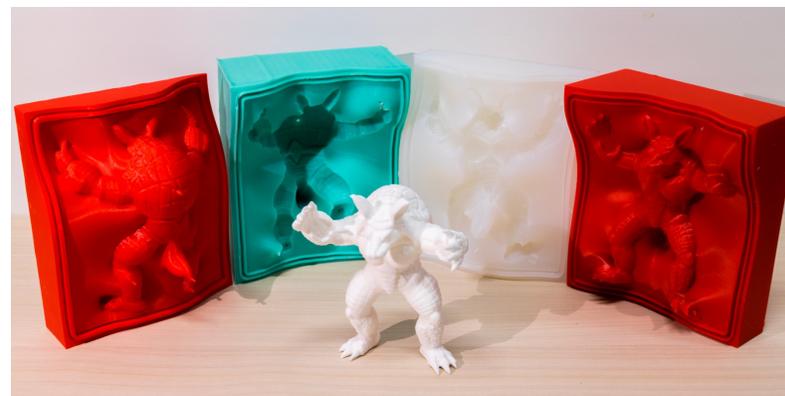
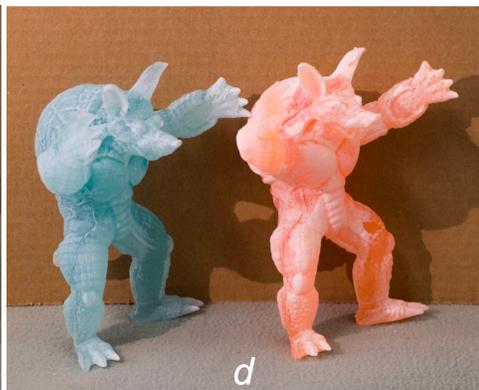
Greedy Labeling for Volume-Aware molding

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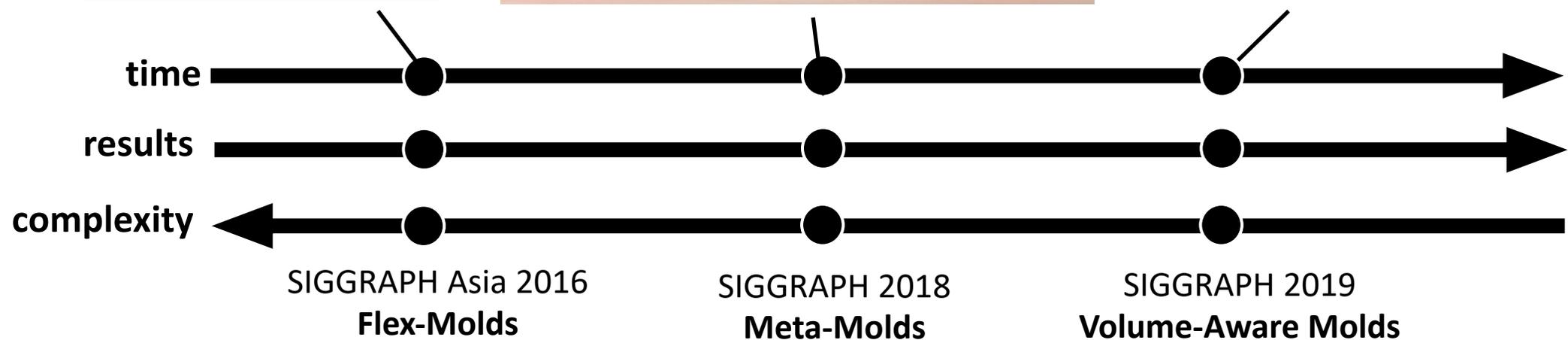
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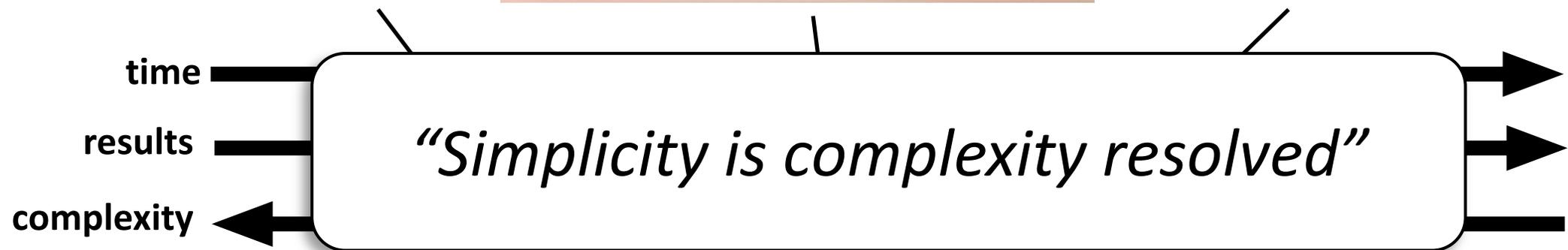
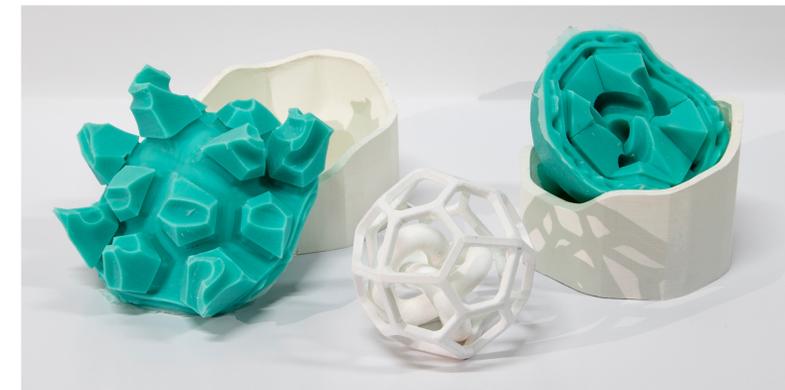
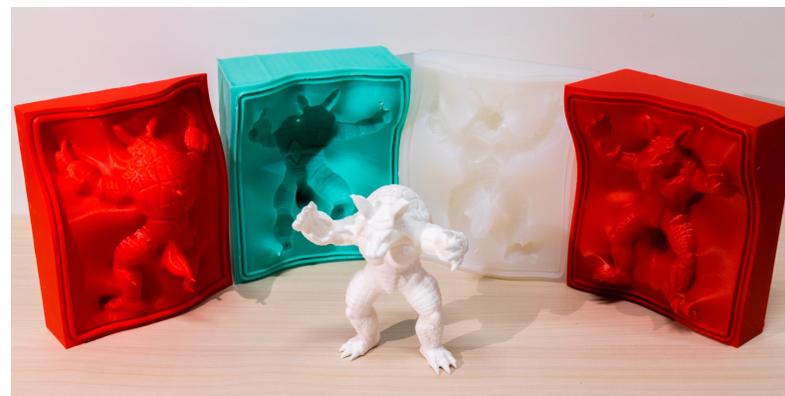
Greedy Labeling for Volume-Aware molding

- Greedy labeling does not typically work as well as graph cuts, but when you have the **right idea**, it becomes extremely powerful!



Greedy Labeling for Volume-Aware molding

- Greedy labeling does not typically work as well as graph cuts, but when you have the **right idea**, it becomes extremely powerful!



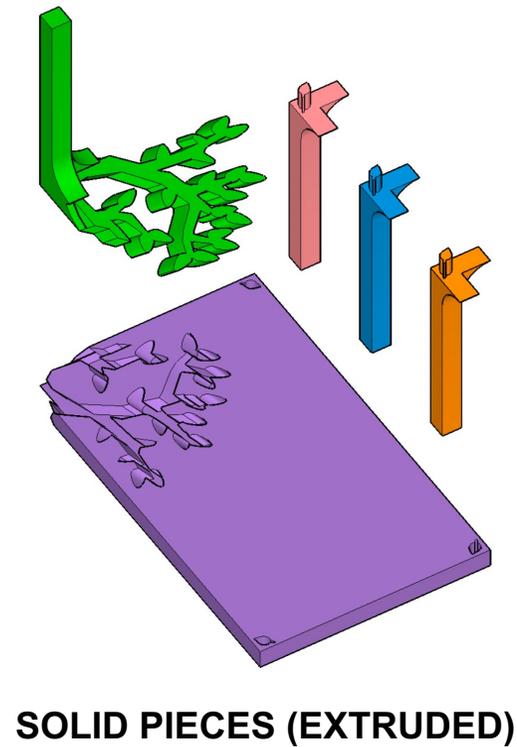
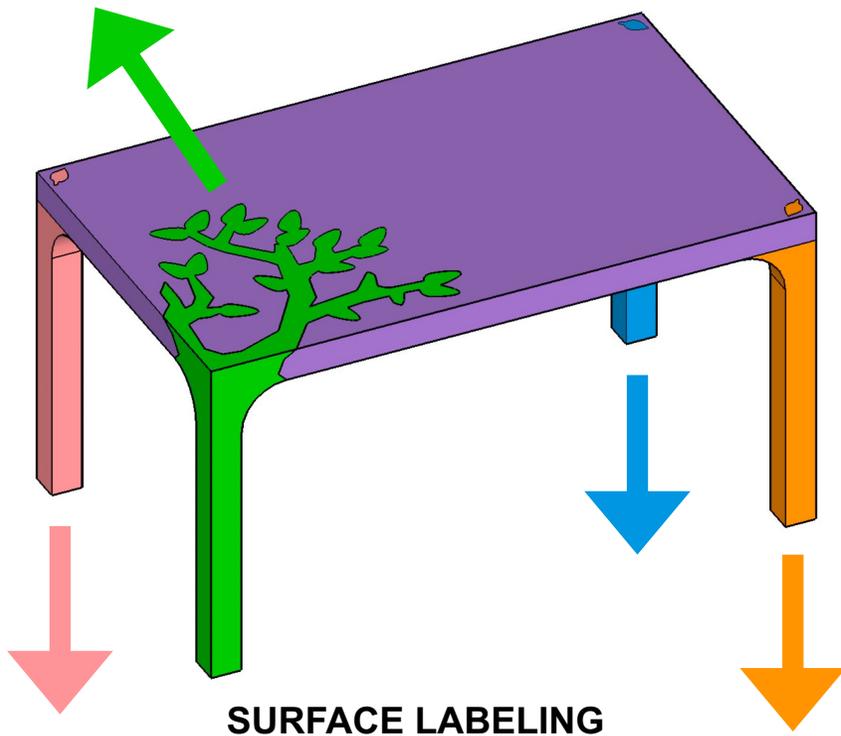
SIGGRAPH Asia 2016
Flex-Molds

SIGGRAPH 2018
Meta-Molds

SIGGRAPH 2019
Volume-Aware Molds

Surface Mesh Booleans

- **Extrusion** of surface patches along a feasible extraction direction



Surface Mesh Booleans

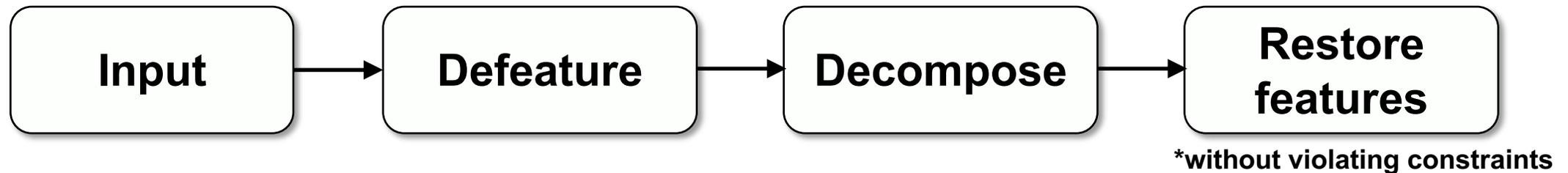
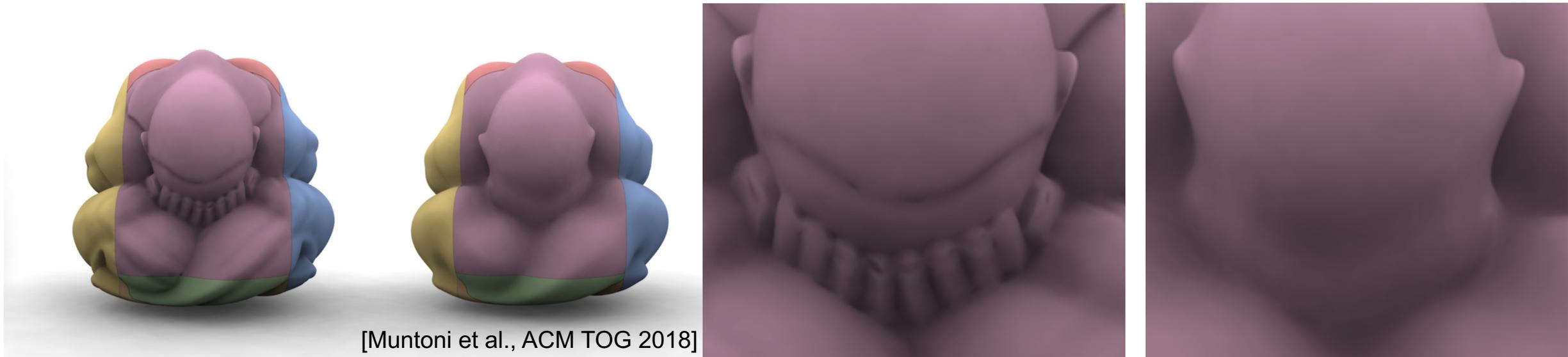
- **Intersection** with an axis aligned box bounding a height field



[Muntoni et al., ACM TOG 2018]

Defeaturing

- Removing small scale features in pre-processing helps reducing part count



Defeaturing

- Removing small scale features in pre-processing helps reducing part count

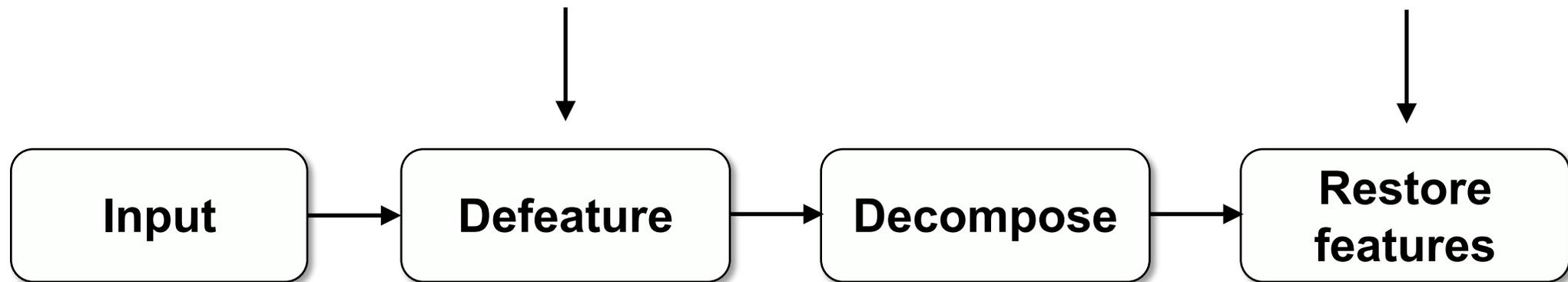
i) Store differential coordinates

$$\delta_i = \frac{1}{|N_i|} \sum_{v_j \in N_i} (v_i - v_j)$$

ii) Apply Smoothing

$$\min \sum_i \|\Delta v_i - \delta_i\|^2$$

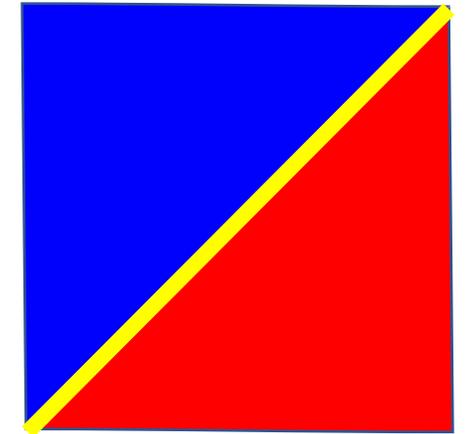
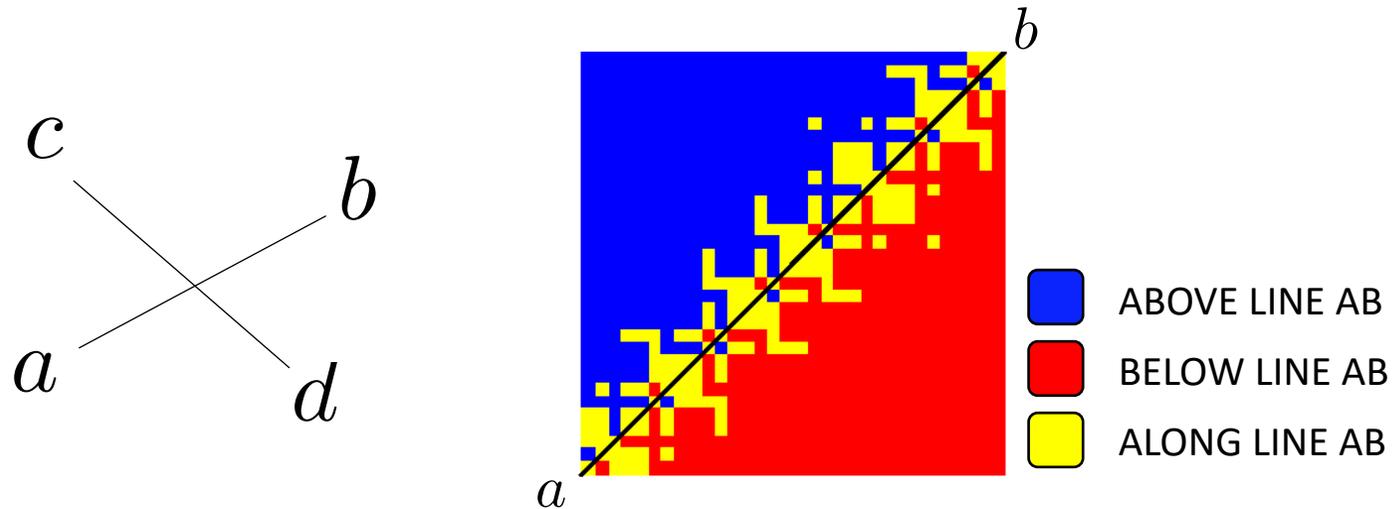
subject to
fabrication constraints



*without violating constraints

Mesh Booleans: Robustness

- Mesh booleans and planar cuts require finding intersections between mesh elements. This computation is **not robust** in floating point!



Indirect Predicates [Attene 2020]
Predicates Construction Kit [Levy 2015]
Shewchuk Predicates [Shewchuk 1997]
GMP/ CGAL

- Rational numbers or exact predicates are robust, but **slow!**
- Labeling (with interface smoothing) is **float friendly**

To Conclude

- Shape Decomposition for Fabrication comes in **many flavours** and is useful for a variety of different things
- It all boils down to control **two basic ingredients**
 - local surface orientation
 - part size (either static or in motion, for assemblability)
- Always a **hard** problem, but we have good heuristics
 - greedy is not a bad word
 - when the idea is good, greedy algorithms become fast, easy to code and powerful!