Supplementary Material for
Conformable Mechanisms on Freeform Surfaces

In this supplementary material, we first provide an example to show the relation between two objective functions E_{motion} and E_{appea} defined in the paper. Next, we provide details of our algorithm to solve the optimization problem in Section 6 of the paper.

1 Relation between E_{motion} and E_{appea}

In the paper, we define two objective functions, E_{motion} and E_{appea}, for designing a conformable mechanism. We provide an example to show that these two objective functions conflict with each other. In detail, for a given reference surface and a target path (represented by 6 points), we randomly sample the search space of a conformable mechanism and visualize the two objective function values of valid mechanism samples (i.e., those satisfying constraints in Section 6 of the paper); see Figure 1. These samples can be approximated by a quadratic regression function, demonstrating that E_{motion} function conflicts with E_{appea} function. This result shows that approximating the target path/motion better usually degrades the mechanism appearance, validating the necessity of using our approach to find a trade-off between these two objectives.

2 Optimization Solver

As described in the paper, we use the Non-dominated Sorting Genetic Algorithm II (NSGA-II) algorithm to solve our optimization problem. A key operation in this algorithm is to evaluate the fitness of a mechanism design candidate. We provide details of this evaluation process in Algorithm 1.

In detail, given a mechanism design candidate with known topology and geometry, we first solve the forward kinematics in Equation 5 of the paper to obtain the joints angles $\mathbf{s}(t)$ at any time $t \in [0, T]$. Then, we compute the maximally allowable driving joint angle Θ to ensure desired mobility by solving Equation 19 in the paper. Next, we verify whether the mechanism satisfies the various constraints defined Section 6 of the paper, as well as evaluate the two objective functions E_{motion} and E_{appea}. We also calculate the sum of residuals, $\mathbf{r}_{\text{residu}}$ for constraints that are not satisfied by the design candidate. After optimizing the mechanism topology and the geometry of links and joints, we further optimize the end effector geometry while fixing its end point position. The end effector geometry is initialized as a straight line, and optimized to minimize E_{gap} defined in Equation 9 of the paper.

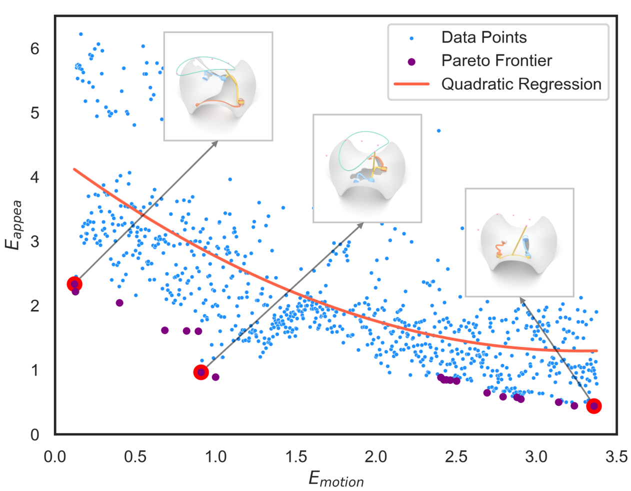

Figure 1: *Visualizing E_{motion} and E_{appea} for sampled conformable mechanisms, giving the same reference surface and target path. We visualize the samples of conformable mechanisms in blue, Pareto frontier of the two objective functions in purple, and quadratic regression function in red. Three example sampled mechanisms are also shown in the figure.*

Algorithm 1 Algorithm to evaluate a conformable mechanism design, taking an enumerated topology *topo*, target points/poses $\{\mathbf{p}_j\}$, reference surface S, desired mobility *mob*, kinematic related parameters $\boldsymbol{\sigma}$, and geometry related parameters $\boldsymbol{\gamma}$ as input. The driving joint angle is assumed to rotate uniformly with angular speed ω. Note that the equations referred in the algorithms are defined in the paper.

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function EVALUATE INDIVIDUAL($topo$, $\{\mathbf{p}_j\}$, S, mob, $\boldsymbol{\sigma}$, $\boldsymbol{\gamma}$)
- $T \leftarrow 2\pi/\omega$
- $\mathbf{s}(t), t \in [0, T] \leftarrow$ solve Eq. 5 with $\boldsymbol{\sigma}$ as input
- $\Theta \leftarrow$ compute driving joint angle range by solving Eq. 19
- $\mathbf{r}_{\text{residu}}$.push_back (residuals of Eq. 13, 15, and 16, given $\mathbf{s}(t), T$)
- $E_{\text{motion}} \leftarrow E_{\text{motion}}(\{\mathbf{p}_j\},\ \mathbf{M}_e(t),\ T)$ in Eq. 10
- $\mathbf{r}_{\text{residu}}$.push_back (residuals of geometry collision constraints in Eq. 14 and 17)
- $\{Tri_k\} \leftarrow$ identify triangles on surface S that collide with moving links and joints
- $S' \leftarrow S \setminus \{Tri_k\}$
- **repeat**
  - $S' \leftarrow$ surface component connected with the driving joint J_0 in S'
  - $S' \leftarrow S' \setminus T_{\text{top-hat}}(S')$ as defined in Eq. 8
- **until** S' does not change
- $E_{\text{appea}} \leftarrow \text{Area}(S \setminus S') + \lambda \sum(\text{Len}(L_i))$ following Eq. 11
- **return** $(E_{\text{motion}},\ E_{\text{appea}},\ \mathbf{r}_{\text{residu}})$

end function
